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A DISSERTATION
SUBMITTED TO THE SCHOOL OF EDUCATION
AND THE COMMITTEE ON THE GRADUATE DIVISION
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

By
Earline Lillian Stoll
March 1962

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Patrick Suppes

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

F. J. McDonald

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Dean of the Graduate Division

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Earline Stoll

Stanford University
March 1962

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CHAPTER I

THEORIES OF CONCEPT FORMATION WITH SPECIAL REFERENCE TO CHILDREN

Concept formation lies at the roots of education. Without a common conceptual framework, communication would be tedious, if not impossible. In an effort to delineate the processes by which concepts are attained, learning theorists have offered a number of explanations and have explored the problem from several experimental directions.

Most of the experiments, however, have used adult subjects. Vinacke has commented that the attainment of a particular concept or group of concepts may be more meaningfully regarded as a process of applying or reorganizing past experience rather than acquiring a "new" concept (Vinacke, 1952). Following this line of thinking, one would not expect the research on concept formation of adults to yield comparable results to those found with children.

Experiments studying concept formation in young children are being conducted by Patrick Suppes and Rose Ginsberg at Stanford University. The first of these experiments examined the learning of binary number concepts by five and six year olds, and it tested specifically the effect of requiring an overt correction response following incorrect responses during learning trials. Earlier experiments with adult subjects have indicated that learning rates are comparable whether or not the subject is required to correct his response overtly following errors; with adults, information indicating which response was correct seems to be sufficient. The results of the experiment with children, however, showed that the correction condition produced faster learning than did the non-correction condition (Suppes and Ginsberg, 1960).

The experiment reported here incorporates many of the features of the Suppes-Ginsberg experiment: presentation of a single stimulus representing an instance of a concept, immediate reinforcement following a correct response, or an overt correction by the subject following an incorrect response. In analyzing the data from their experiment, Suppes and Ginsberg noted that many of their empirical observations closely approximated those predicted by a stimulus sampling model developed by Bower (Bower, 1961). It was felt that the rigor engendered by the model provided a more precise test of the theoretical assumptions than was possible without the model. Since the experiment reported here is similar to the Suppes-Ginsberg one, it has been designed in a manner that will permit the application of the same learning model to the data.

Bower had initially applied this model to paired-associate learning. The basic notion of the model is that each stimulus item in the list of paired-associates is represented by a single stimulus element within the model and that the correct response to that item becomes conditioned in an all-or-none fashion. A single item is either conditioned or not conditioned to the correct response. All items are originally unconditioned and become conditioned through reinforcement. The effect of repeated reinforcements of the correct response to a given stimulus item is to provide repeated opportunities for the all-or-none association to be acquired.

One of the interesting facets of this "one-element" model is the treatment of errors. If an error occurs on trial n̲, the item has not been conditioned prior to that trial. Hence it can be assumed that at the beginning of trial n̲ the subject's associative connection has not effectively changed since the start of the experiment and that what has happened on the earlier trials has no effect on the subject's behavior after trial n̲.

The experiment reported here provides a situation in which a single stimulus pattern is presented to the subject on each trial. Originally the experiment was designed to test only whether the concept is learned on an all-or-none basis. The stimuli were to be presented in random order without any cues other than the feedback from reinforcement and correction. However, the pilot study indicated that no learning took place under these conditions. The author became interested in whether the number of dimensions defined for the subject would affect the learning rate. Consequently the experiment was changed to permit the examination of the effect of a simultaneous presentation of a relevant dimension of the concept. In this case, the relevant dimension was defined by a small outline figure (model) representative of the concept to be learned. It was decided to vary the number of dimensions defined for the subjects. Those dimensions defined for a particular subject were indicated by the presence of the models behind the appropriate response key on the testing apparatus. There are no data available on the effect such models have on concept learning. But it was felt that a figure representing a generalized form of the concept might provide a cue for classification.

The experiment was, therefore, designed to investigate (1) the relationship between the efficiency of learning simple concepts and the number of dimensions defined for the subject (i.e., the number of models present when the concept is presented) and (2) the question of whether or not simple concepts are learned in an all-or-none fashion.

Since kindergarten presents children the earliest opportunity for organized learning experience in the public school, it was felt that pupils of this level provided an appropriate background for a study of the stimulus variables which influence concept learning when new concepts are being learned. Geometric form and angle size were chosen because they are among the simplest of the spatial concepts and are frequently encountered in the child's environment. In light of the research on the effect of feedback, the author chose to test the children in a situation involving immediate reinforcement and correction, an overt correction being required following an incorrect response.

Specifically, the experiment was designed to test the following hypotheses: (1) the rate at which kindergarten children learn concepts of geometric form and angle size is related to the number of representative models present at the time the stimulus is presented; (2) simple concepts of geometric form and angle size are learned in an all-or-none fashion.

CHAPTER II

DESIGN OF THE EXPERIMENT

Subjects: Eighty-eight kindergarten children, with an average age of 5.5 years, were tested individually. All the pupils in three kindergarten classes were used. The order of testing subjects depended upon a randomization of the names in the teacher's register. All three classes were heterogeneous in socioeconomic background. No attempt was made to group the children according to IQ; so it may be assumed that they were heterogeneous in that respect also.

Apparatus: The testing apparatus was designed so that the subject faced a window on a board before him, below which was a row of three white buttons and a row of three colored lights (red, green, and amber). The upright portion of the apparatus measured 15" x 18" with a window 10½" x 8¼". The lower portion extended forward 7", the lights being placed 2" from the upright board and the buttons 2" in front of the lights. When the model figures were used, they were placed on the upright portion directly behind and slightly above the appropriate response key. Squares of construction paper (3" x 3") were placed behind the response keys for which there was no model figure exhibited.

Stimuli: Two sets of stimulus cards (8½" x 11"), each set containing 153 geometric figures, were used. One set contained polygons, divided into three subsets of 51 each: triangles, quadrilaterals, and pentagons. The other set of 153 figures contained angles, equally divided into three subsets of acute angles, obtuse angles, and right angles. A given subject learned only one set of cards. Each subset of polygons included figures representative of the various types within the specific subset: the triangles included isosceles, equilateral, right, scalene acute, and scalene obtuse triangles; the quadrilaterals included squares, rectangles, rhombuses, parallelograms, trapezoids, and trapeziums; the pentagons included both regular and irregular figures. All polygons were convex. The acute angles were angles of 15^o, 30^o, 45^o, 60^o, and 70^o; the obtuse angles were 110^o, 120^o, 135^o, 150^o, and 165^o. The area of each polygon and the length of sides of each angle were determined by five basic figures: three squares (3", 4", 5") and two rectangles (3½" x 4½", 3" x 4"). The polygons and angles were randomly assigned to the basic figures and inscribed within them. Thus, although area varied from one figure to the next, it was an irrelevant cue and was not appropriate as a basis of classification of the figures. The orientation of the figures was rotated randomly; thus, orientation was also an irrelevant cue. The use of a range of shapes and angle sizes minimized the effect of classification according to a special case of the figure. The complete set of 153 cards was randomized in 17 blocks of nine with the requirement that each block contain three cards from each subset; e.g., each set of nine in the polygon group included three triangles, three quadrilaterals, and three pentagons.

Pilot Study: Preliminary to the main experiment, 30 children were tested: 18 children from the first grade, four from the kindergarten, four from the second grade, and four from the third grade. The original plan of the experiment called for no models of figures, but the results of the pilot study indicated that the children did not learn under these conditions. The length of each testing session was also varied during this period of exploration. Best results were obtained with a test session of approximately 15 minutes. Therefore, the main experimental sessions were limited to six blocks of stimuli (a total of 54) with a maximum number of four sessions for each child.

Procedure: Each subject was exposed to only one of the two sets of stimulus cards (angles or forms). Each subset within the set (e.g., obtuse, right, or acute angles) corresponded to one of the three buttons, the assignment being randomized for each subject to counterbalance the effect of position and color preference.

The 88 subjects were randomly assigned to eight experimental conditions (groups). Four groups learned the polygon problem and four learned the angle problem. The experimental variable was the number of model figures available above the subject's response keys. For example, if a model of an acute angle were to be present for a given subject, then a typical member of this class was printed in black outline on a 3" x 3" card and displayed immediately above the correct response key for the class of acute angles. Different subjects learned with 0, 1, 2, or 3 models present, the number of subjects being 16, 16, 24, and 32, respectively, in the four conditions, half learning the angle problem and half the polygon problem. In the one-model and two-model conditions the subset to which the model (or models) belonged was varied; i.e., for some subjects the model was a triangle, for others a pentagon or a quadrilateral. The number of subjects under the different conditions was varied when it became evident that there was little learning under the no-model and one-model conditions. The hypothesis being tested was that speed of learning would be directly related to the number of model figures available to the subject during learning.

For each set of stimulus cards there were eight random orders presented to the first eight subjects in each groups, four additional randomizations to be presented to the subjects nine through 12 in the two-model and three-model groups, and four additional randomizations to be presented to subjects 13 through 16 in the three-model groups. Each subject received the same set of instructions before the experiment. He was told that he was going to play a game in which the pictures he saw belonged to one of the pictures (or colors) on the board. He could win the game by matching the picture presented with the correct model. When a picture was presented, the child responded by pressing the button in front of the picture (or color) to which he believed the larger picture belonged. If his response was correct, the light behind the button he pressed flashed. If he was incorrect, the light in front of the correct picture (or color) flashed, and he was required to press the correct button before the stimulus was removed. There was no time limit for exposure of the stimulus, but the average exposure time was between 10 and 15 seconds. The subjects received 54 trials during days one and two, 45 on day three (in order to complete the set of 153 cards), and 54 on day four. The stimuli presented on day four were randomly chosen blocks from the stimuli presented earlier. A learning criterion was set at one errorless block of nine trials (the minimum trials being set at 27), and a subject's learning series was terminated whenever his performance reached this level.

The experimenter was partly concealed behind the testing apparatus and responded to the child only when necessary to maintain the child's interest in the experiment. Each child was asked at the end of the polygon problem to count five objects; since the polygon classification depended upon the number of vertices (3, 4, or 5), it was of interest to see whether the non-solving subjects could count five objects.

CHAPTER III

RESULTS OF THE EXPERIMENT

In testing the hypothesis that the speed of learning concepts of this type is related to the number of model figures present, the proportions of correct responses for each of the eight groups were calculated by dividing the total number of correct responses by the total number of responses made. Sixteen graphs of average learning curves are presented in the following pages.

Figures 1 and 2 represent the correct responses for the subjects receiving the same sequence under the four conditions (eight subjects under each condition) -- Figure 1 for the polygon groups and Figure 2 for the angle groups. Figures 3 and 4 show the learning curves for the subjects receiving the same sequence under the two-model and three-model conditions, 12 subjects under each condition. Eight of the subjects in Figures 3 and 4 are included in the learning curves for Figures 1 and 2. It was of interest to note whether the responses of the additional four subjects in the two-model and three-model groups changed the shape of the learning curves shown in Figures 1 and 2. Figure 5 presents the learning curves for the subjects under the three-model condition (16 in each group) -- one curve for the polygon group and one for the angle group. Figures 6 and 7 describe the average learning curves for all subjects under the four conditions -- Figure 6 for the polygons and Figure 7 for the angle groups. There were eight subjects under both the no-model and the one-model conditions, 12 subjects under the two-model condition, and 16 under the three-model condition. Figures 8 and 9 present separately the learning curves for the three concepts learned by the three-model group: Figure 8 shows the proportion of correct responses for the triangle, quadrilateral, and pentagon; Figure 9 provides similar information for the acute, right, and obtuse angles. Figures 10 and 11 describe for the two-model groups the same kind of separation as Figures 8 and 9 describe for the three-model group. Figures 12 and 13 contrast the correct responses of the two-model groups under two conditions: (1) the proportion of correct responses for the concept not represented by a model figure and (2) the proportion of correct responses for the concepts represented by models. Figure 12 contrasts the correct responses for the triangle under the two-model condition: (1) when there was a model figure present and (2) when there was no model present. Figures 15 and 16 present for the one-model groups the same kind of contrast as Figures 12 and 13 present for the two-model groups.

The differences represented in Figures 1 and 2 were tested for statistical significance. Table 1 presents the results of an analysis of variance of correct responses among the four polygon groups receiving the same sequence of stimuli and among the four angle groups receiving the same sequence. The analysis of variance tested whether the four model conditions (0, 1, 2, or 3 models) produced significant differences in speed of learning as measured by the number of correct responses. Comparisons between model conditions were made at two points: on the first block of nine trials and on the total responses (through the twenty-third block of nine trials). Table 2 presents a summary of t tests comparing certain means computed in the analysis of variance for Table 1.

In testing the hypothesis that the formation of concepts of geometric form and angle size in young children follows the all-or-none model, the data were analyzed on the three-model group of 16 subjects. This group was chosen for analysis because it included only subjects who reached criterion and hence were considered to have learned. (Three subjects who failed to reach criterion, one in the angle group and two in the polygon group, were replaced by other subjects who attained criterion. These replacements did not change the statistical significance of differences between groups learning under the different conditions. It was felt that these replacements were justified since we were interested only in subjects who had learned the concepts before trial 207).

Using the assumption that no learning takes place before the last error, only the responses before the last error were analyzed. The probability of an error was calculated for each concept by dividing the total number of errors (before the last error) by the total number of responses before the last error. These probabilities were used to test the assumptions that no learning takes place before the last error and that, prior to conditioning, the subject guesses at random in responding to the stimuli.

The assumption that the subject guesses at random before conditioning implies that the responses prior to the last error should be essentially like the successive outcomes of tossing a coin that comes up "success" a certain proportion of the time. It is also assumed that successive responses (coin tosses) are independent; that is, the outcome on trial n + 1 does not depend on the outcome of the prior trials n, n - 1, etc. Such a series of independent trials is called a "Bernoulli" series.

To determine whether the responses before the last error form a Bernoulli series, three goodness-of-fit tests were used, each testing for different aspects of a Bernoulli series: (1) the stationarity test which tested the hypothesis that there is no change in the proportion of correct responses over trials (i.e., that there is no improvement in performance -- learning -- before the last error); (2) the order test which tested the independence in the responses made from trial to trial (i.e., whether the probability of a success is the same following a success or failure on the preceding trial); (3) the test to determine whether the number of correct responses in blocks of four trials follows a binomial distribution (since it is known that the sum of four Bernoulli random variables has a binomial distribution). All three tests were chi-square tests, each testing the null hypothesis that the observed distribution did not differ significantly from the theoretical distribution provided by the model. The calculations were made separately for each concept except in the following cases: the data from the subset of triangle stimuli were not analyzed because there were too few errors to provide sufficient numbers for analysis; the data on the quadrilateral and pentagon were analyzed first separately and then combined; stationarity tests were performed on the data for each angle (acute, right, obtuse) and also on the combined data from all three angles.

For the stationarity test, testing the assumption that there was no gradual improvement in performance over trials before the last error, the responses (before the last error) for each concept were summed over the 16 subjects and then analyzed in blocks of six trials. The total number of errors in each block was divided by the total number of responses to determine the probability of an error. An overall chi-square test was made on the first five blocks for each concept. After five blocks, the responses were too few to yield reliable estimates of error probabilities. The formulation of the relevant chi-square test is found in Suppes and Ginsberg (1961). The results are tabulated in Table 3.

To show the stationarity of the guessing probabilities prior to conditioning, Figures 17-21 present curves of the proportion of correct responses prior to the last error. The mean learning curves for all responses are presented in the same figures, including the responses for those who have reached criterion for a particular concept.

For the order test, testing the assumption that there is statistical independence in the responses from trial to trial prior to the last error, a count was made for each concept to determine whether there was a significant difference in the order of responses; i.e., tabulations were made for error following error, success following error, error following success, and success following success. A chi-square test was used to determine the statistical significance of the empirical findings. The appropriate chi-square test is presented in Suppes and Ginsberg (1961). The results are tabulated in Table 3.

For the binomial distribution test, the responses before the last error for each of the 16 subjects were symbolized by 0 (indicating error) and 1 (indicating success) for each concept and then marked off in blocks of four trials. Distributions were tabulated for each concept according to the number of errors in each block; i.e., the total number of blocks having 0, 1, 2, 3, 4 errors, respectively, for a particular concept was summed over all subjects in the group. A chi-square test was applied to the distribution for each concept (except the triangle) to test the assumption that the responses are binomially distributed. The results are tabulated in Table 3.

Further formulations which were made to determine how well the model predicted the observed data in this situation included: (1) the standard deviation of errors per concept per subject, (2) the expected errors before the first success, (3) the standard deviation of errors before the first success, (4) the expected number of error runs, (5) the expected error runs of length 1, (6) the expected error runs of length 2, (7) the expected number of alternations of success and failure, (8) the expected pairs of errors one trial apart, (9) the net difference between error following error and error following success. The formulas used for these predictions were derived in Bower's report (1961) and in the Suppes and Ginsberg report (1960). The conditioning parameter $\underline{c}$ used in these formulas was derived for each concept on the basis of the guessing probability for that particular concept. The probability of guessing correctly was 1 - q, where q = the total number of errors prior to the last error divided by the total number of responses prior to the last error. The results are presented in Tables 4 and 5.

TABLE 1

ANALYSIS OF VARIANCE OF NUMBER OF CORRECT RESPONSES COMPARING THE FOUR MODEL CONDITIONS, EACH F-VALUE HAVING THREE AND TWENTY-EIGHT DEGREES OF FREEDOM

	Polygon Problem		Angle Problem	
	Trials 1 - 9	Trials 1 - 207	Trials 1 - 9	Trials 1 - 207
F-value	4.06	19.10	3.74	19.94
P<	.025	.001	.04	.001

TABLE 2

t TESTS FOR COMPARING GROUP MEANS IN THE DATA ANALYZED IN TABLE 1[*]

Type of Stimuli	Trials in Test	Models for Group	t	P<
Polygons	1 - 9	0 and 2	.82	Non-sig.
Polygons	1 - 9	0 and 3	3.12	.01
Polygons	1 - 9	1 and 3	2.86	.01
Polygons	1 - 9	2 and 3	2.21	.04
Polygons	1 - 207	0 and 1	.42	Non-sig.
Polygons	1 - 207	0 and 2	2.61	.02
Polygons	1 - 207	1 and 2	2.19	.05
Polygons	1 - 207	2 and 3	4.13	.001
Angles	1 - 9	0 and 1	.79	Non-sig.
Angles	1 - 9	0 and 2	1.58	Non-sig.
Angles	1 - 9	2 and 3	1.59	Non-sig.
Angles	1 - 9	0 and 3	3.19	.01
Angles	1 - 9	1 and 3	2.39	.03
Angles	1 - 207	0 and 1	1.6	Non-sig.
Angles	1 - 207	0 and 2	3.94	.001
Angles	1 - 207	2 and 3	3.33	.01
Angles	1 - 207	1 and 2	2.34	.05

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[*] For testing the difference between these means, the S^2 was S_w^2 derived in the analysis of variance. The *df* for each *t* was 28 (McNemar, 1955).

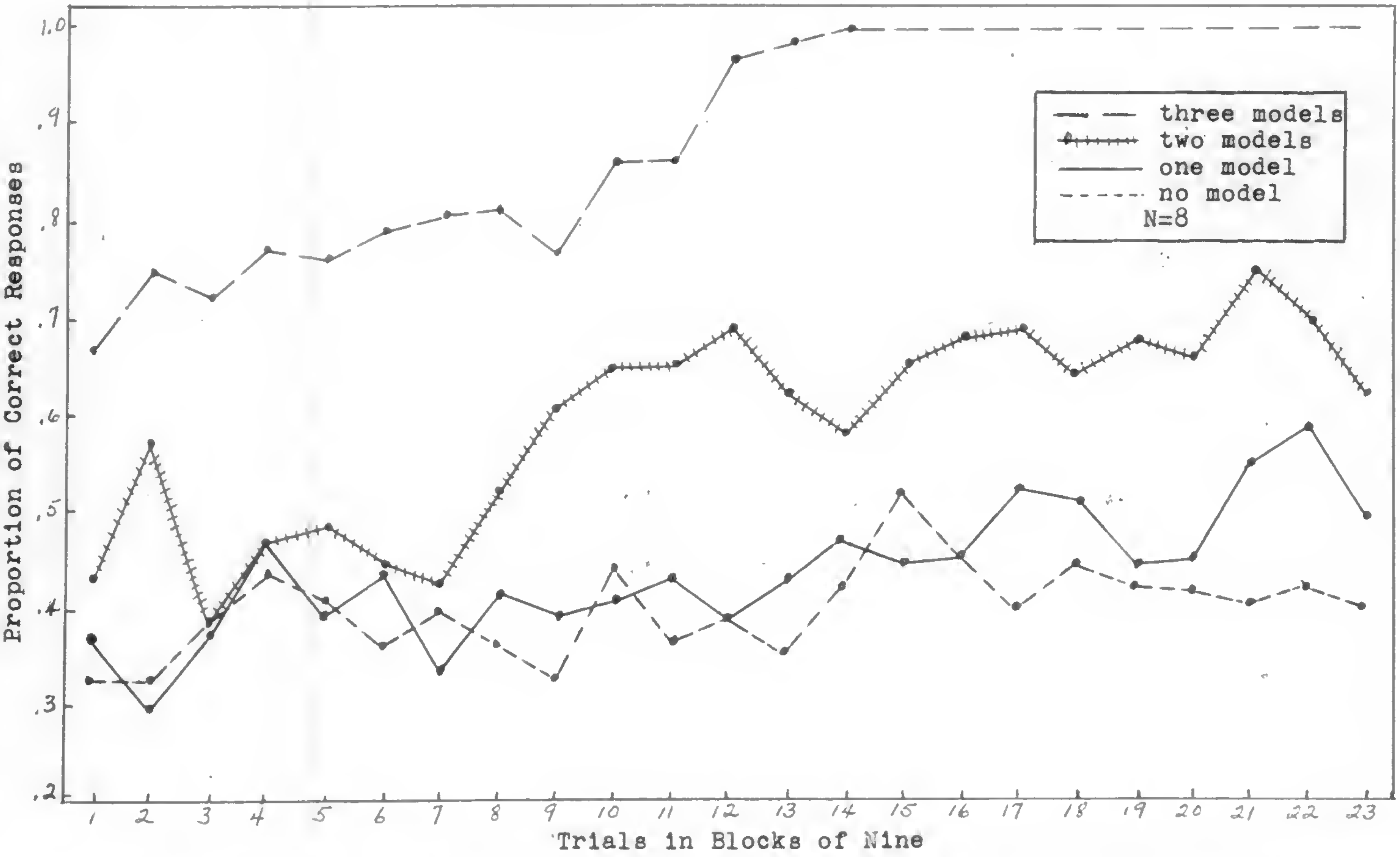

Fig. 1. Mean learning curves for polygons under the four conditions of model presentation.

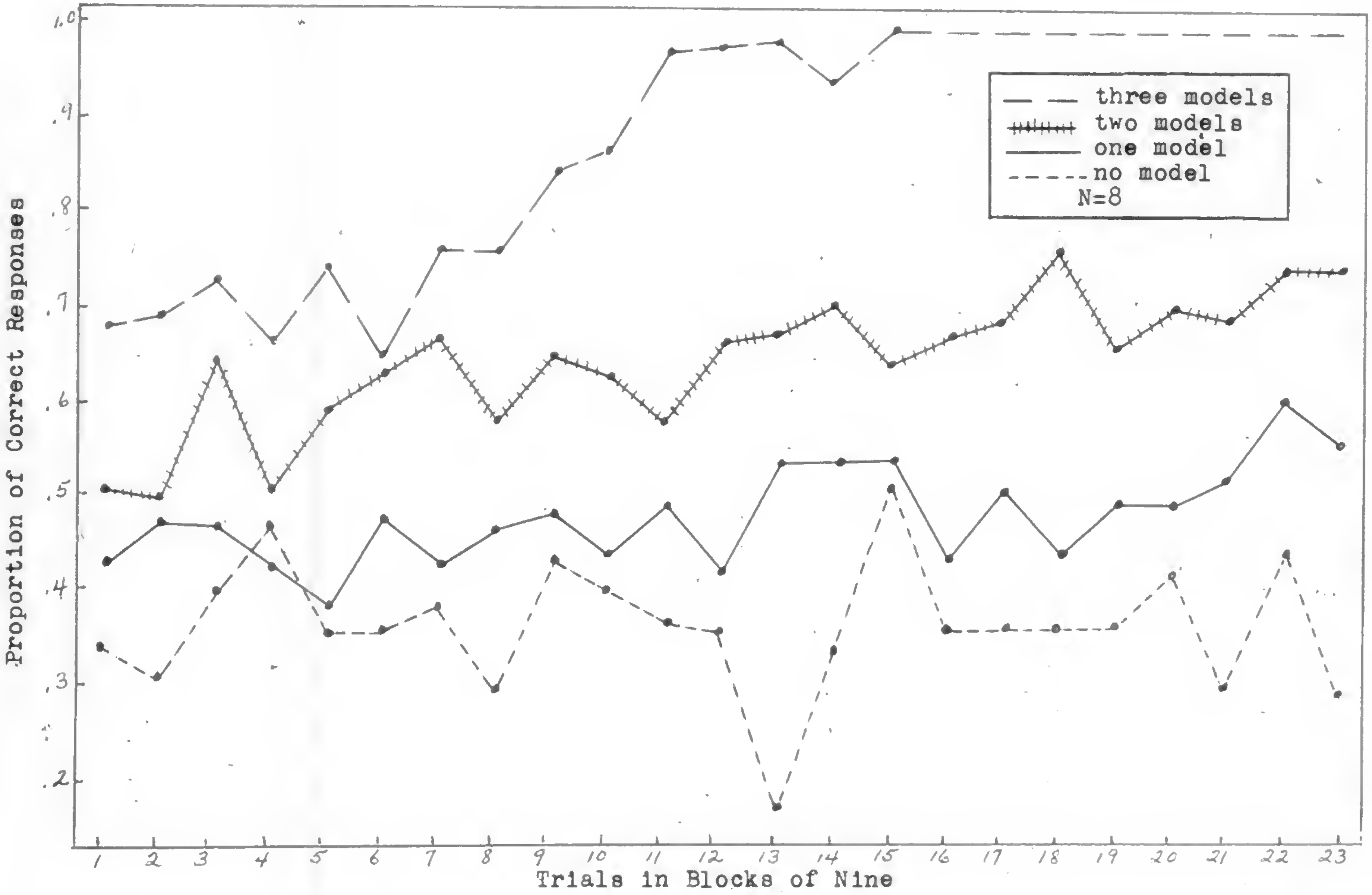

Fig. 2. Mean learning curves for angles under the four conditions of model presentation.

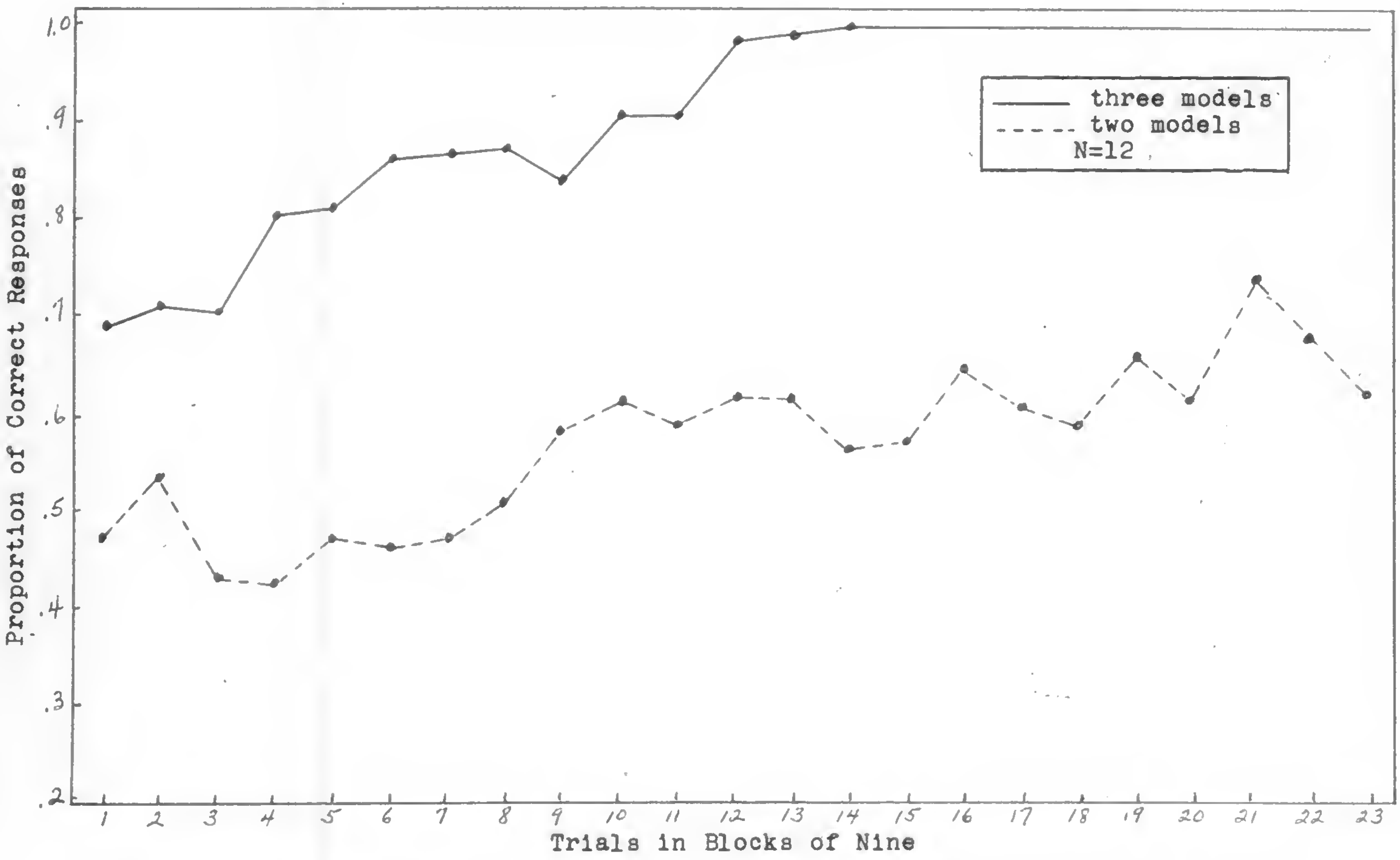

Fig. 3. Mean learning curves for polygons under two conditions of model presentation.

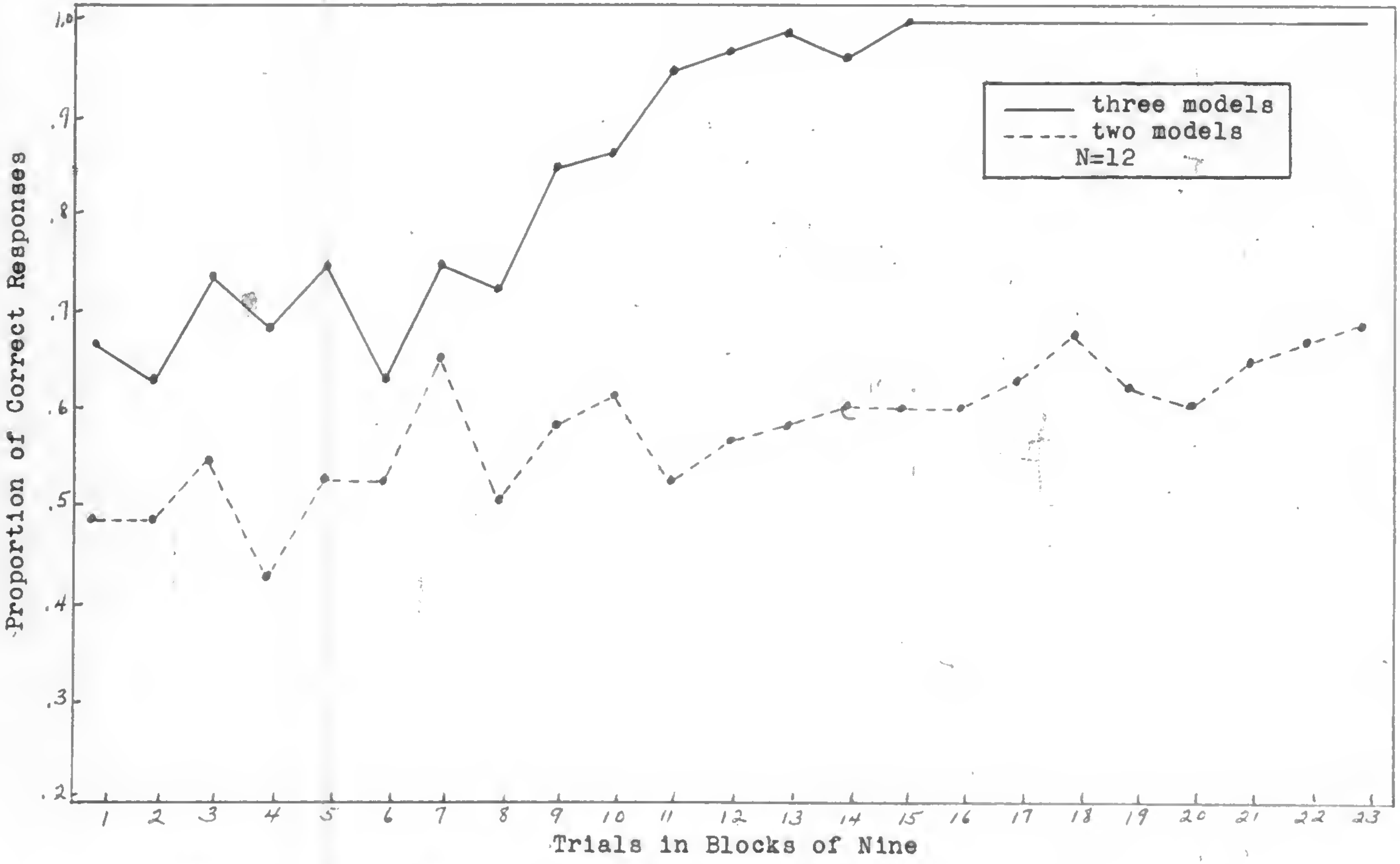

Fig. 4. Mean learning curves for angles under two conditions of model presentation.

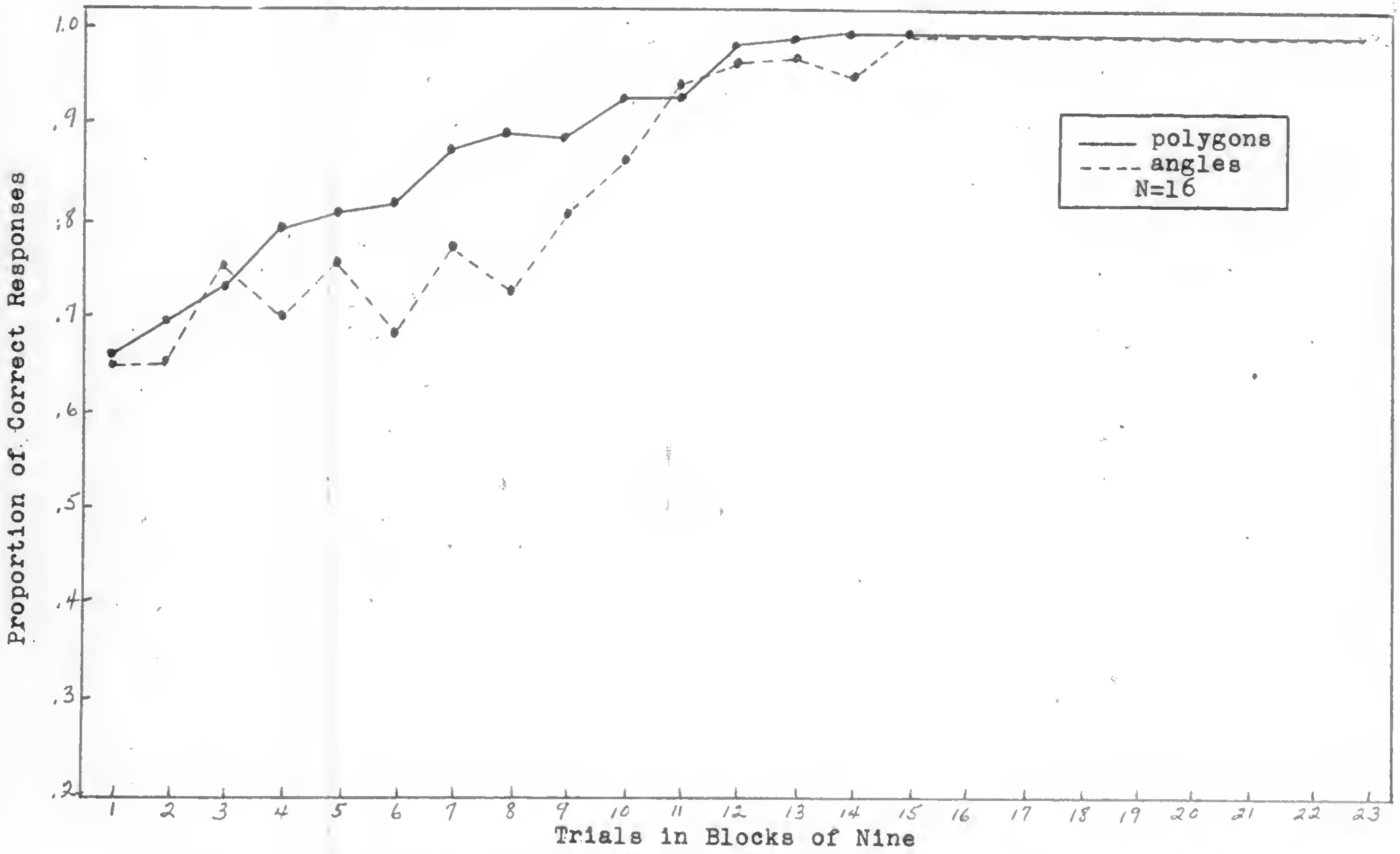

Fig. 5. Mean learning curves for the polygon and angle groups under the conditions of three-model presentation.

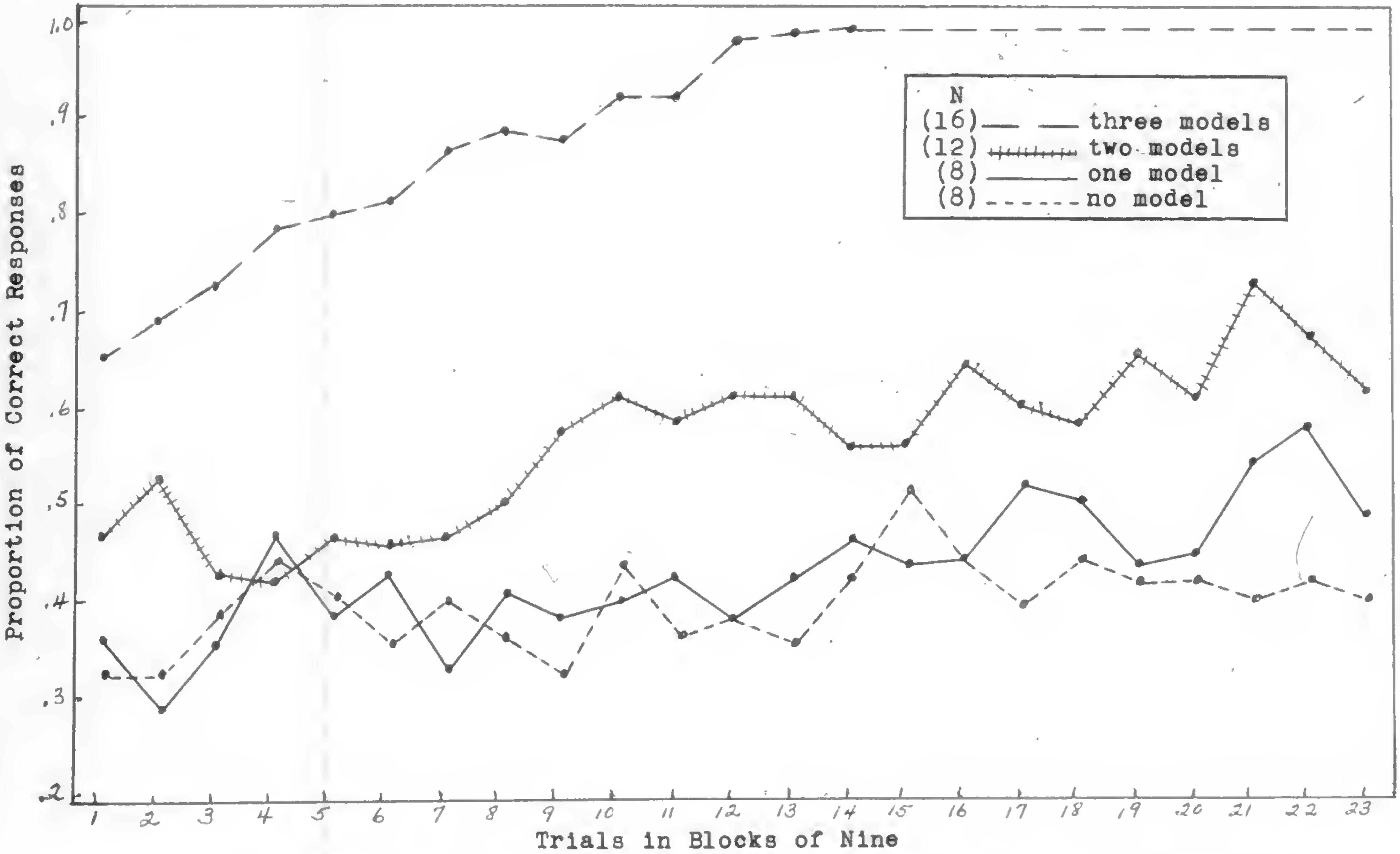

Fig. 6. Mean learning curves for polygons under the four conditions (using all subjects).

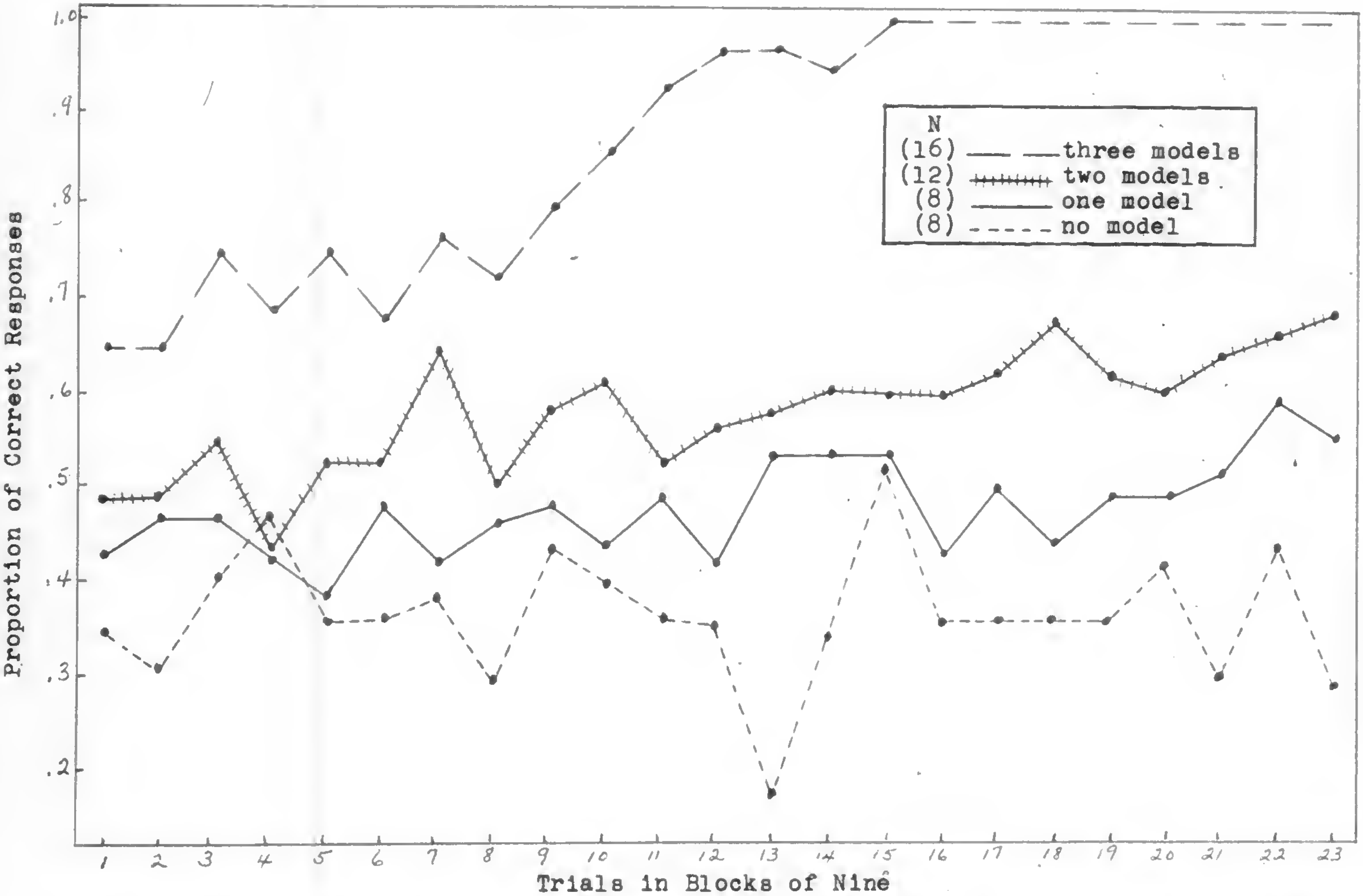

Fig. 7. Mean learning curves for angles under the four conditions (using all subjects).

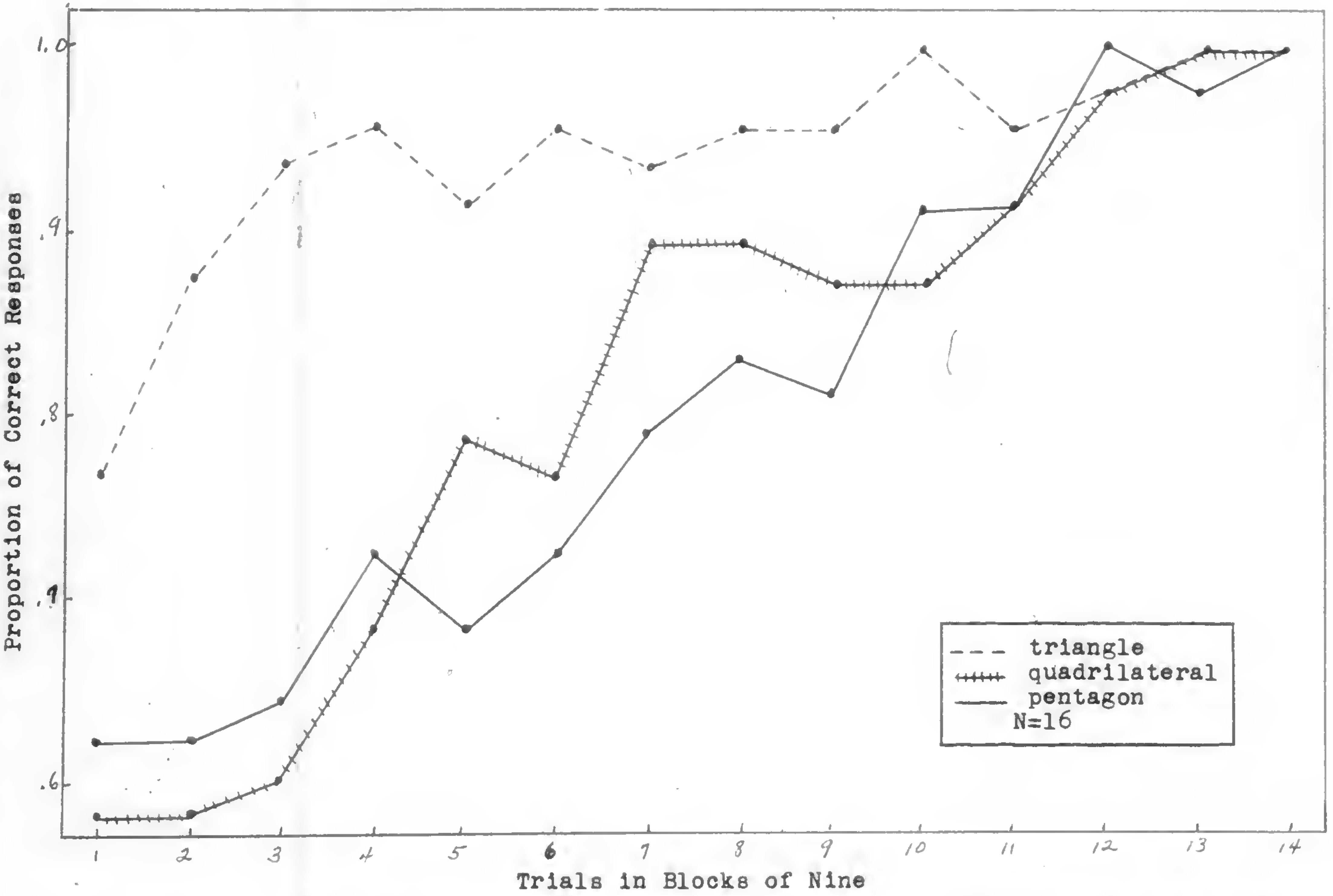

Fig. 8. Mean learning curves for the three polygon concepts under the three-model condition.

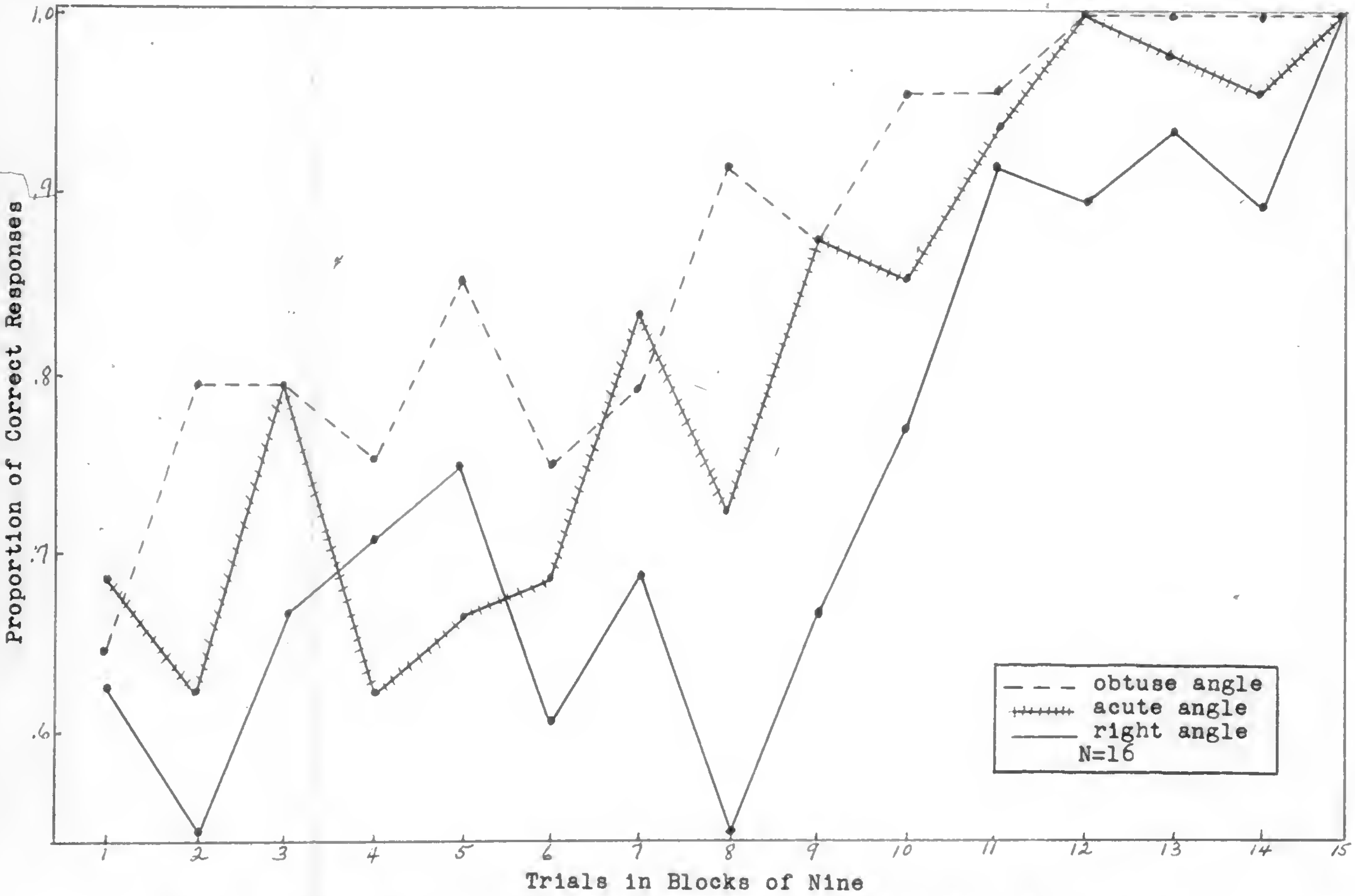

Fig. 9. Mean learning curves for the angle concepts under the three-model condition.

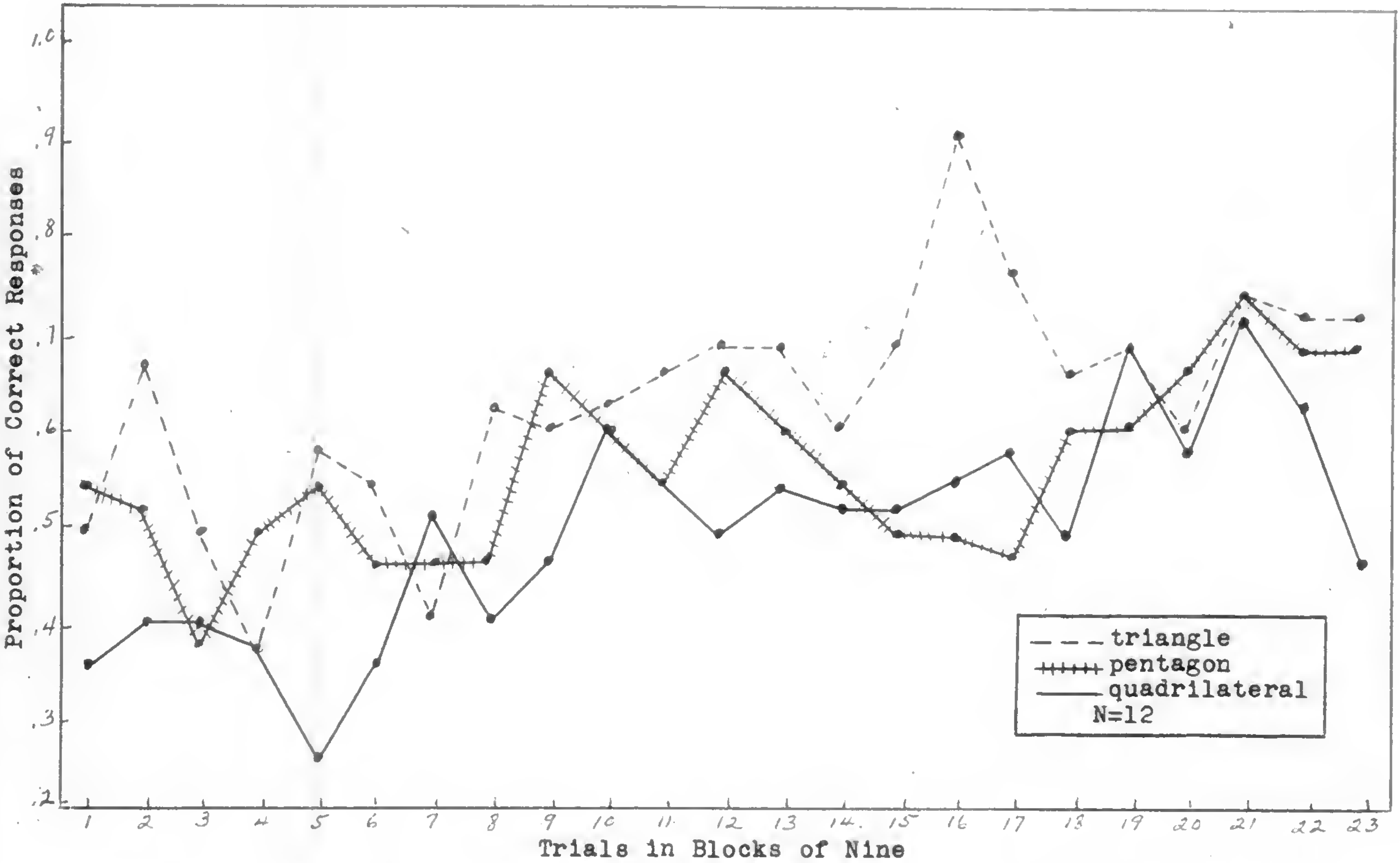

Fig. 10. Mean learning curves for the polygon concepts under the two-model condition.

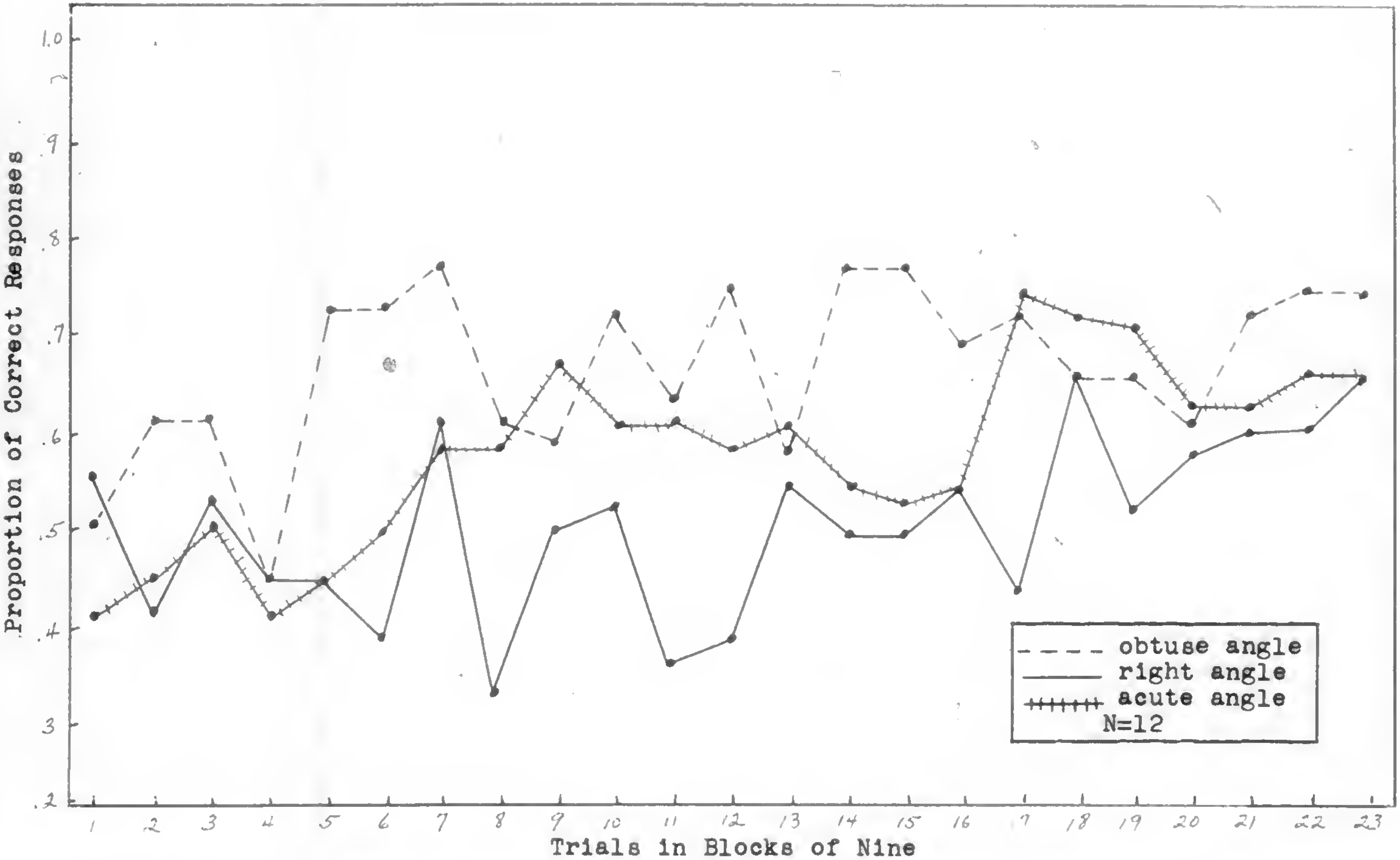

Fig. 11. Mean learning curves for the angle concepts under the two-model condition.

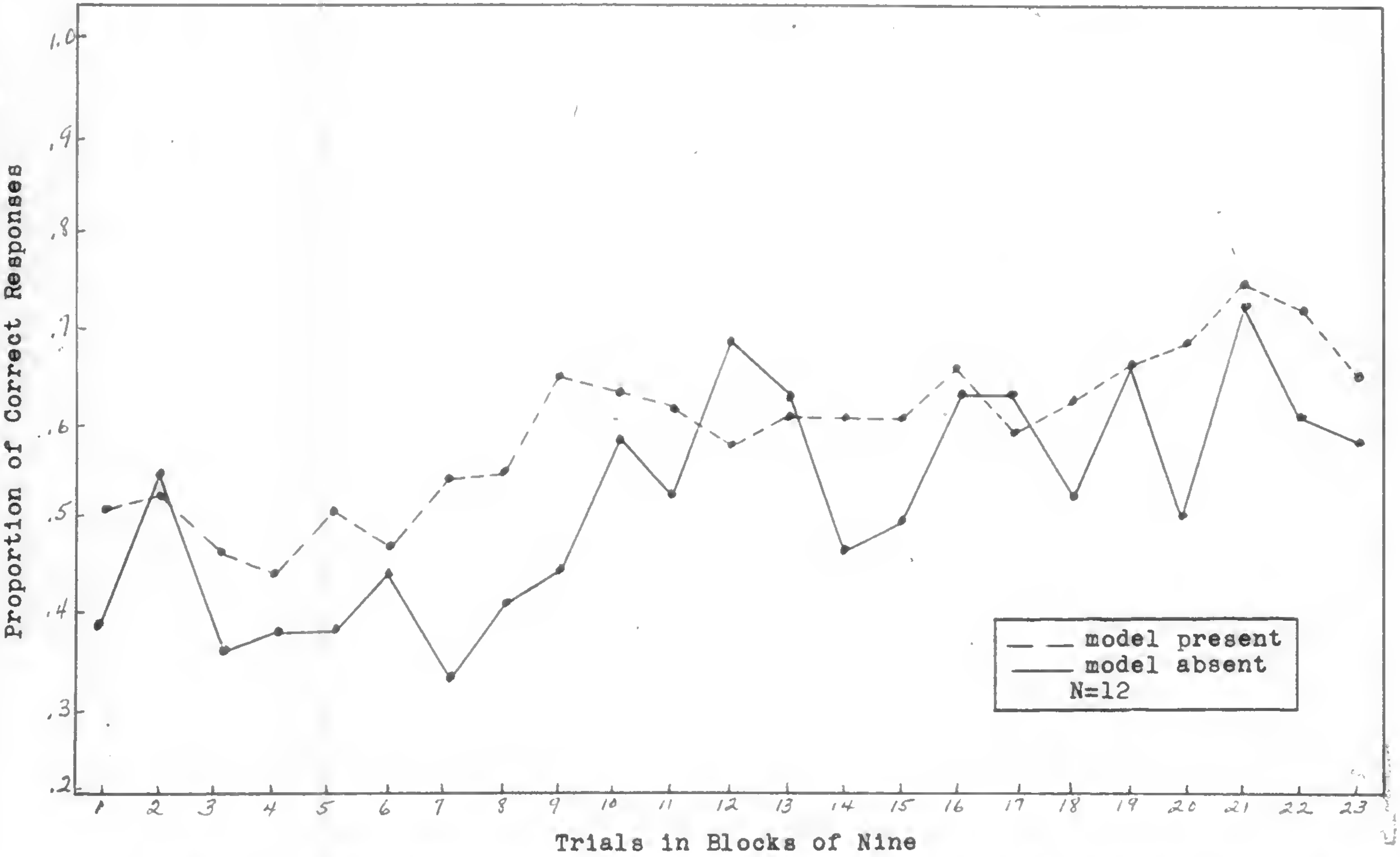

Fig. 12. Mean learning curves for the polygons under the two-model condition, model present and model absent.

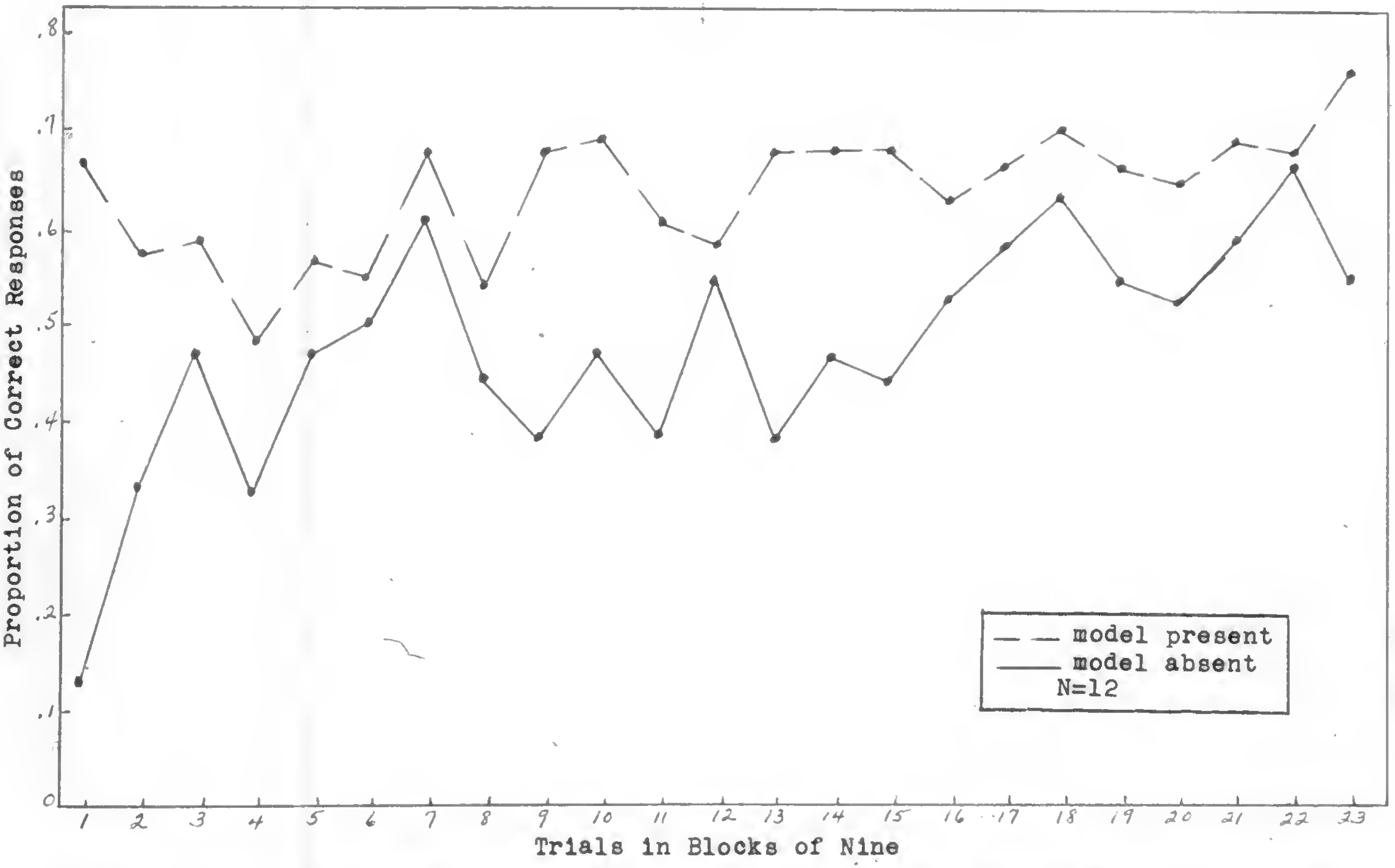

Fig. 13. Mean learning curves for the angles under the two-model condition, model present and model absent.

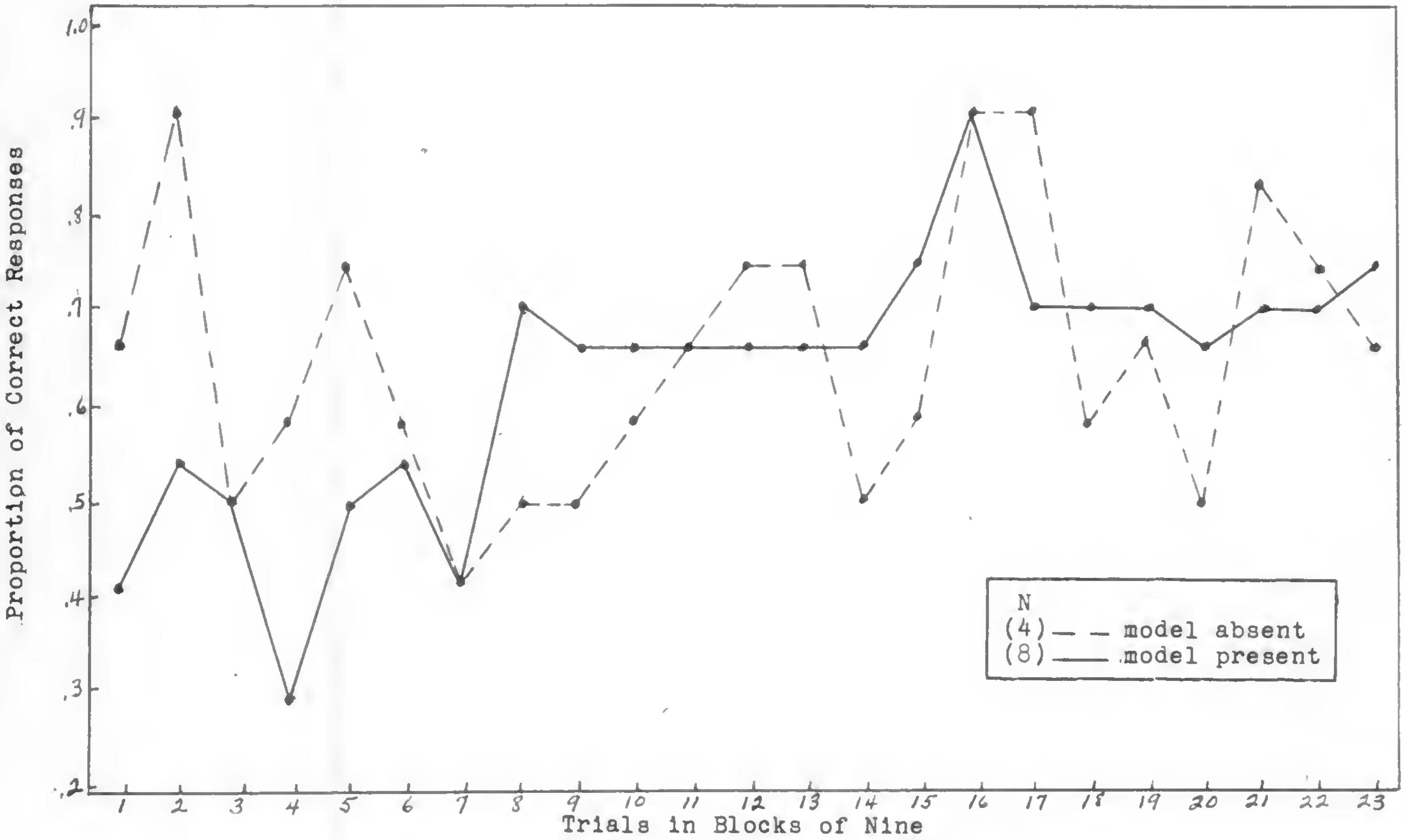

Fig. 14. Mean learning curves for the triangle concept under the two-model condition, model present and model absent.

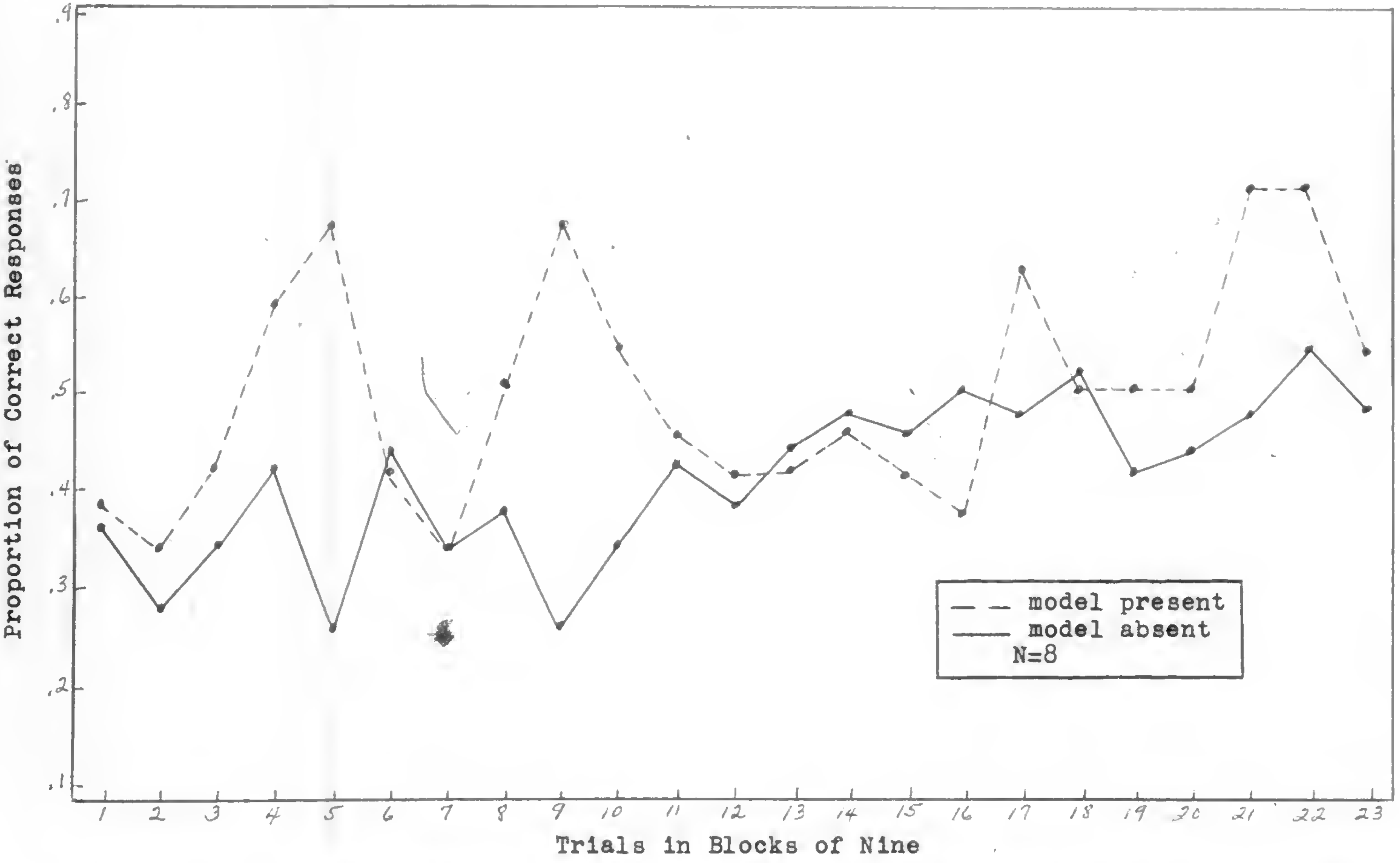

Fig. 15. Mean learning curves for the polygons under the one-model condition, model present and model absent.

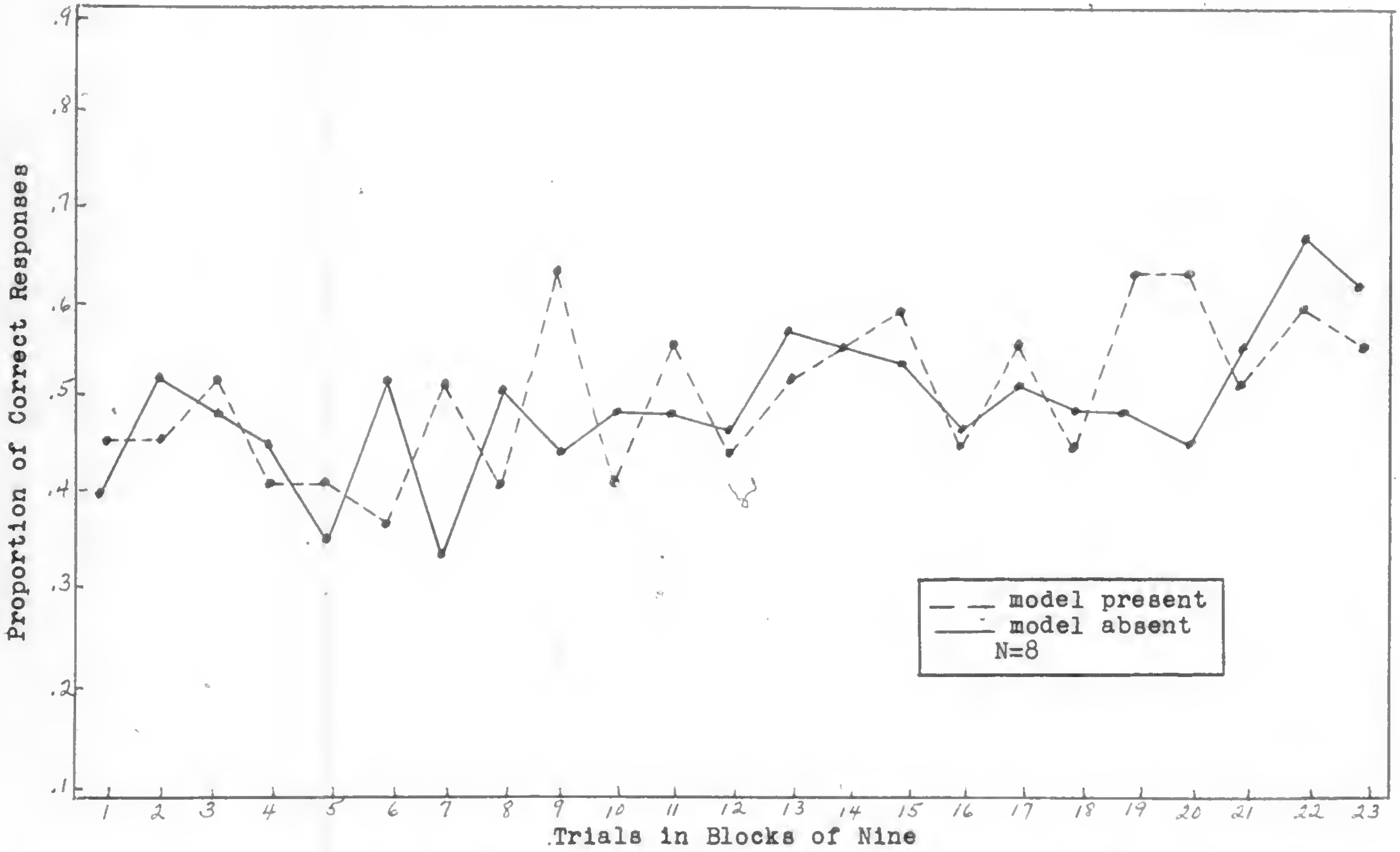

Fig. 16. Mean learning curves for the angles under the one-model condition, model present and model absent.

TABLE 3

STATIONARITY, ORDER, AND BINOMIAL DISTRIBUTION RESULTS

Concept and Test	Chi-square	df	P >
Quadrilateral, p = .619			
Stationarity (N = 273)	1.68	4	.70
Order (N = 262)	.65	1	.40
Binomial Distribution (N = 65)	.923	2	.60
Pentagon, p = .600			
Stationarity (N = 275)	2.40	4	.60
Order (N = 269)	1.76	1	.15
Binomial Distribution (N = 65)	2.07	2	.35
Acute Angle, p = .674			
Stationarity (N = 338)	7.96	4	.05
Order (N = 348)	3.17	1	.05
Binomial Distribution (N = 85)	2.66	2	.25
Right Angle, p = .506			
Stationarity (N = 313)	6.34	4	.10
Order (N = 326)	2.41	1	.10
Binomial Distribution (N = 80)	10.52	2	.001*
Obtuse Angle, p = .721			
Stationarity (N = 268)	1.10	4	.85
Order (N = 256)	7.32	1	.001*
Binomial Distribution (N = 63)	2.90	2	.20
Quadrilateral and Pentagon, p = .609			
Stationarity (N = 548)	.71	4	.90
Binomial Distribution (N = 130)	1.77	2	.40
All Angles, p = .624			
Stationarity (N = 919)	.97	4	.90

*Significant

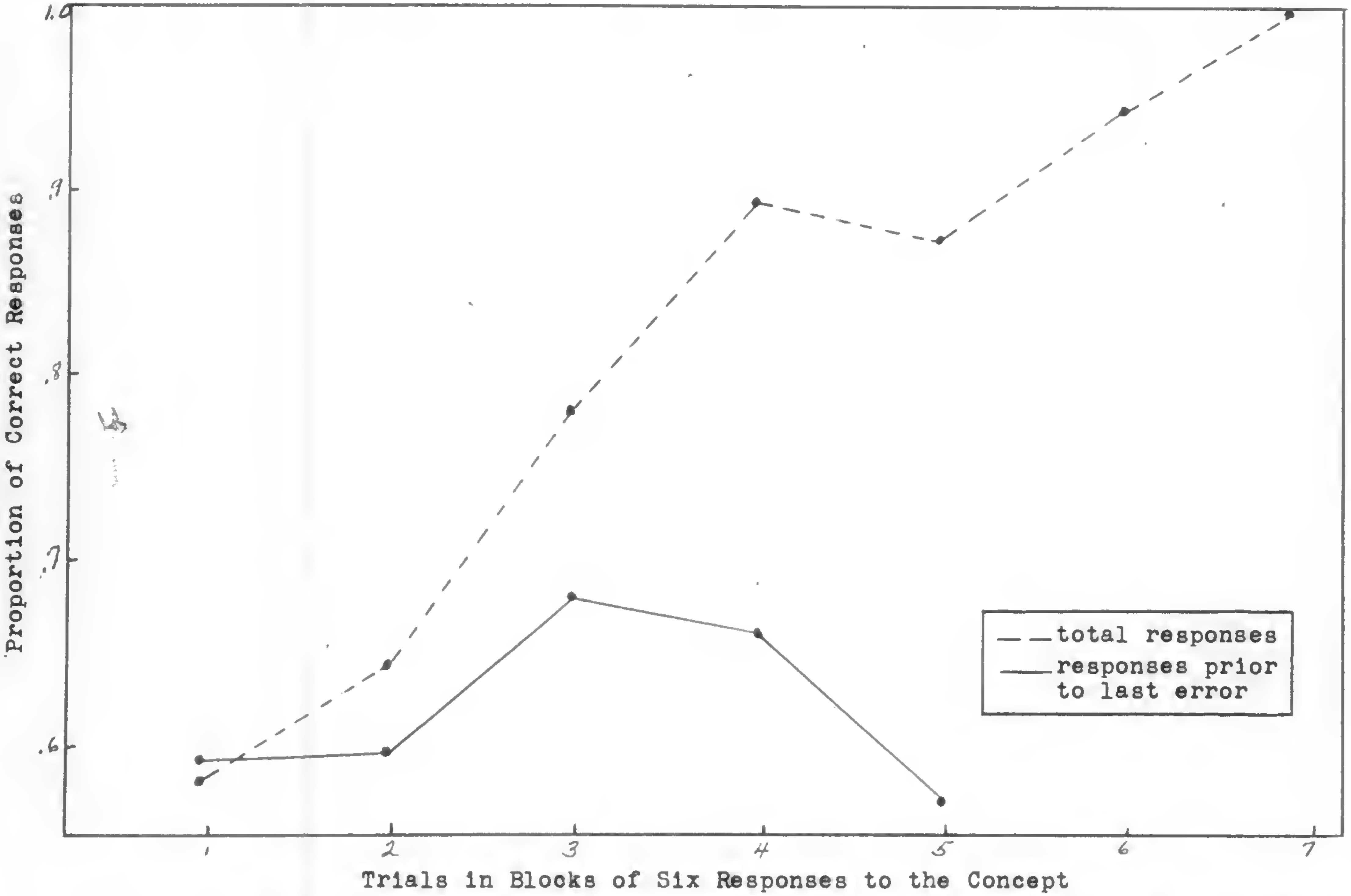

Fig. 17. Proportion of correct responses prior to the last error and the mean learning curve for the quadrilateral.

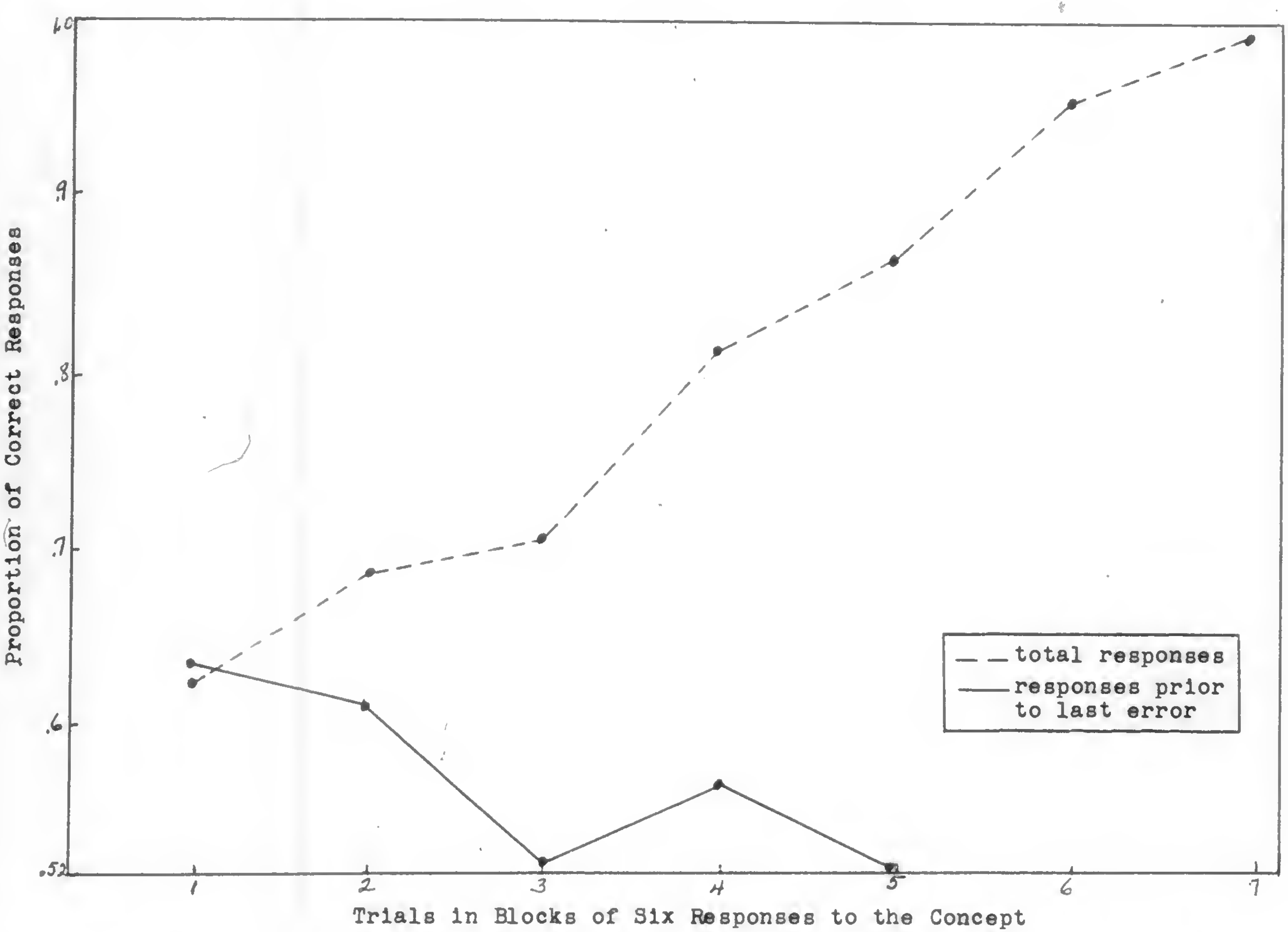

Fig. 18. Proportion of correct responses prior to the last error and the mean learning curve for the pentagon.

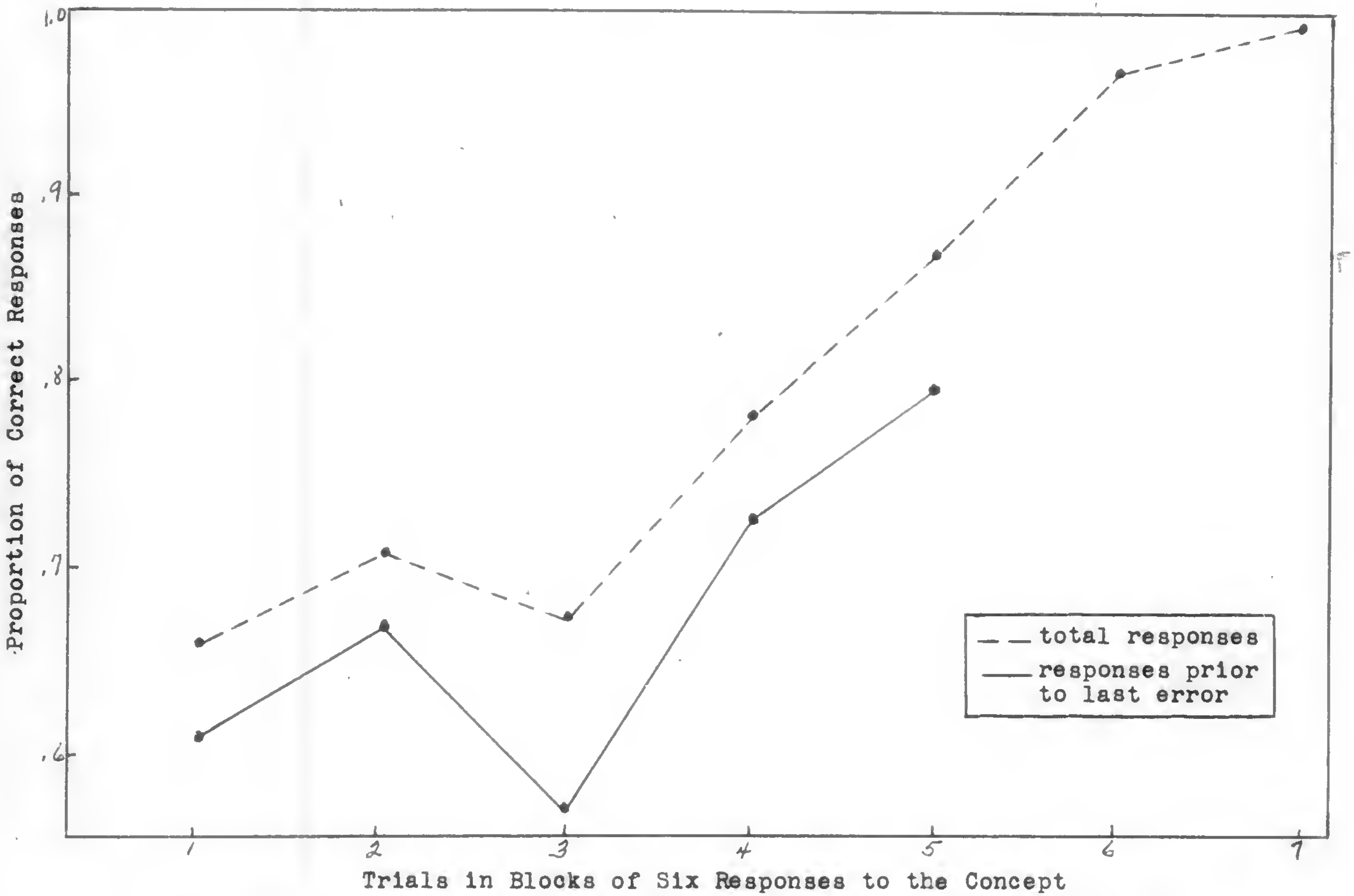

Fig. 19. Proportion of correct responses prior to the last error and the mean learning curve for the acute angle.

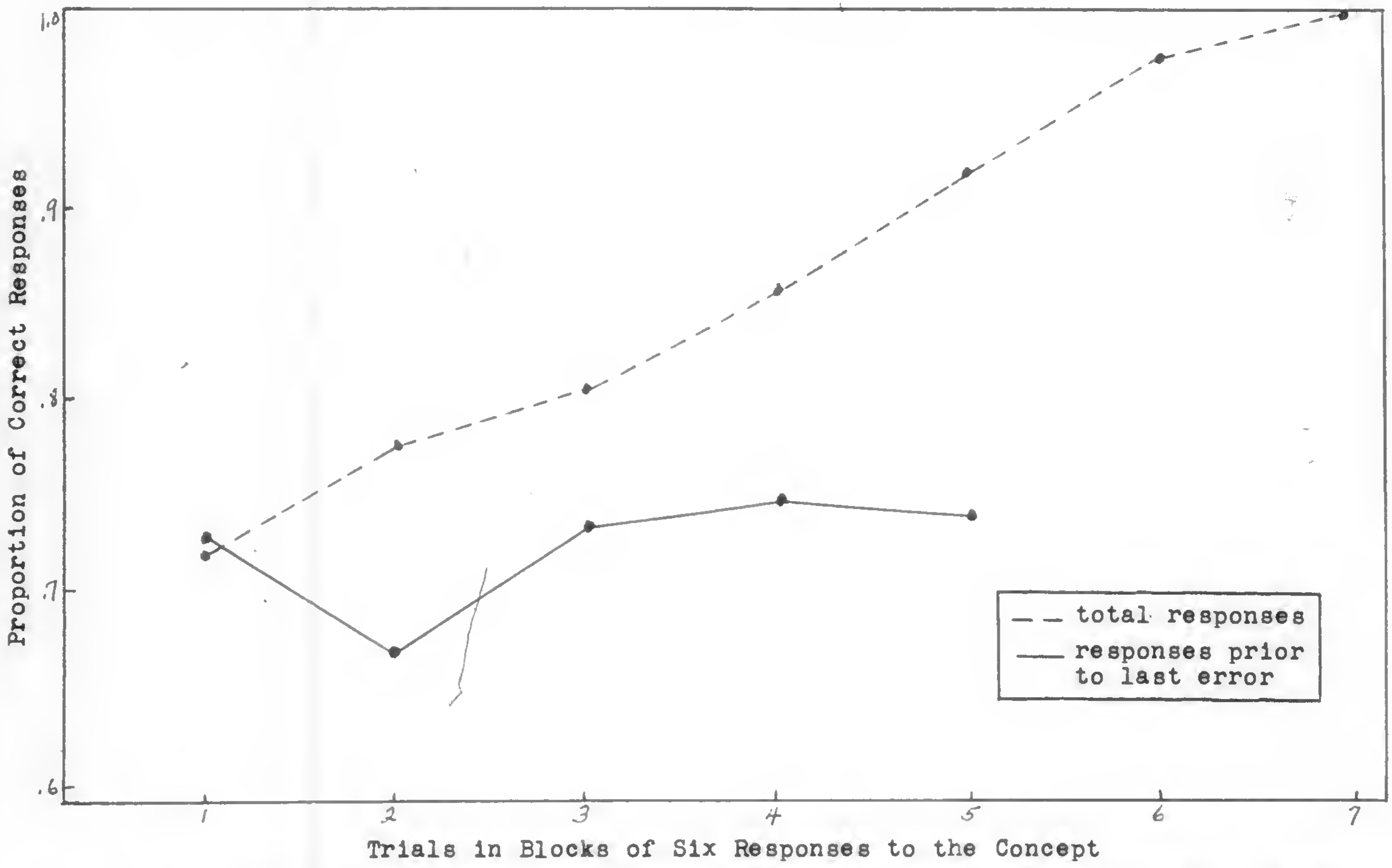

Fig. 20. Proportion of correct responses prior to the last error and the mean learning curve for the obtuse angle.

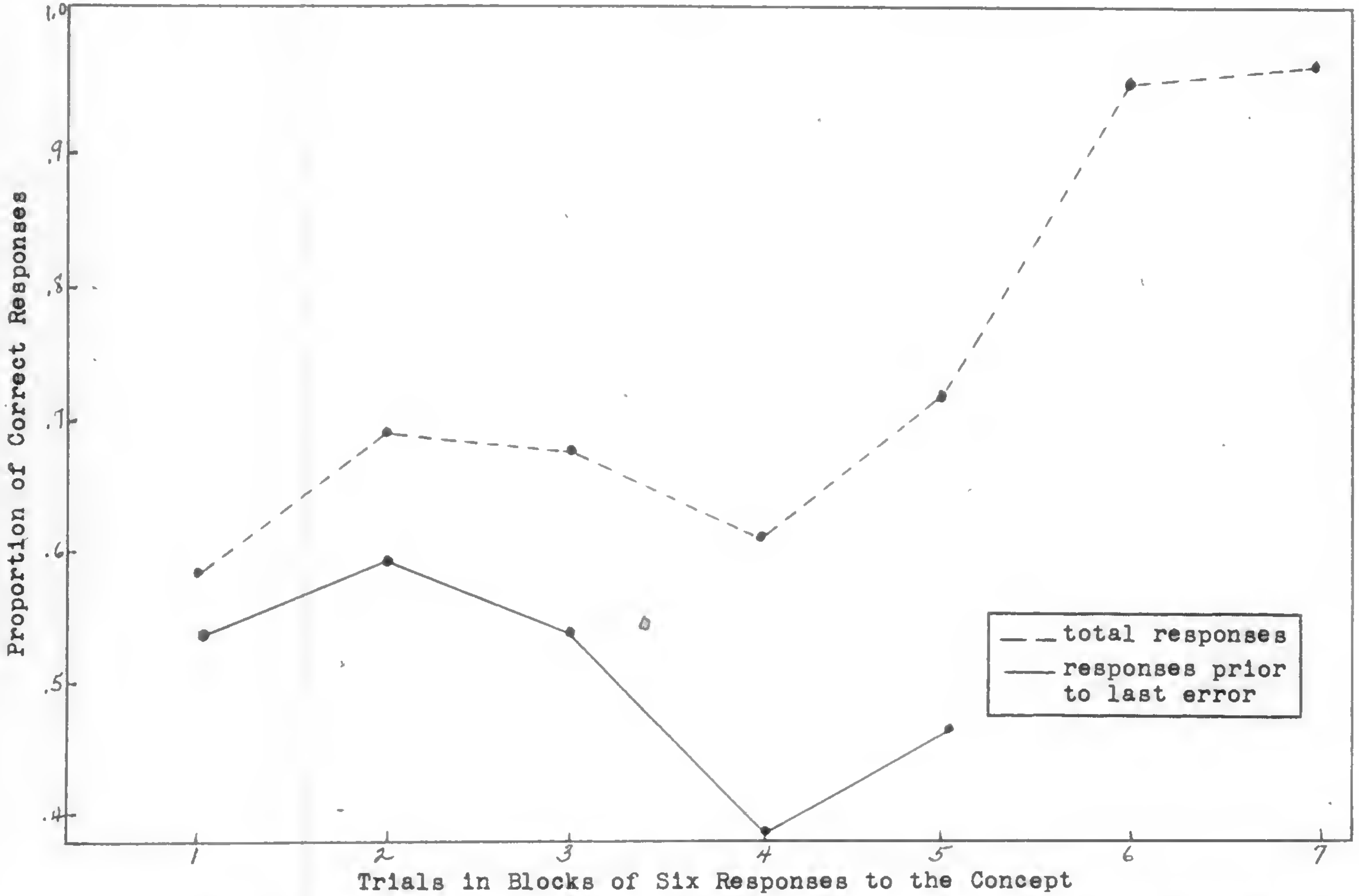

Fig. 21. Proportion of correct responses prior to the last error and the mean learning curve for the right angle.

TABLE 4

STATISTICS USED IN CALCULATING THE PREDICTIONS IN TABLE 5

Problem	Mean Total Errors E(t)	Prob. of Correct Guess 1-q	Estimate of Learning Rate, c
Triangle	2.375	.78	.0926
Quadrilateral	7.625	.62	.0498
Pentagon	8.125	.60	.0492
Acute Angle	8.250	.67	.0400
Right Angle	11.375	.51	.0431
Obtuse Angle	5.750	.72	.0487

TABLE 5

PREDICTED AND OBSERVED STATISTICS

Statistic	Predicted	Observed
Standard deviation of expected errors per item per *S*:		
Triangle	2.64	2.62
Quadrilateral	7.74	3.92
Pentagon	8.23	5.64
Acute Angle	8.42	4.75
Right Angle	11.38	8.94
Obtuse Angle	5.97	4.64
Errors before first success:		
Triangle	.28	.31
Quadrilateral	.61	.44
Pentagon	.65	.56
Acute Angle	.48	.31
Right Angle	.92	1.13
Obtuse Angle	.38	.44
Standard deviation of errors before first success:		
Triangle	.58	.58
Quadrilateral	.99	.93
Pentagon	1.05	.93
Acute Angle	.84	.58

TABLE 5 -- Continued

Statistic	Predicted	Observed
Right Angle	1.30	2.03
Obtuse Angle	.72	.86
Total number of error runs:		
Triangle	1.90	2.12
Quadrilateral	4.87	4.56
Pentagon	5.04	4.68
Acute Angle	5.64	5.25
Right Angle	6.04	5.56
Obtuse Angle	4.22	3.68
Expected error runs of length 1:		
Triangle	1.52	1.87
Quadrilateral	3.11	2.25
Pentagon	3.12	2.93
Acute Angle	3.85	3.25
Right Angle	3.21	2.87
Obtuse Angle	3.10	2.62
Expected error runs of length 2:		
Triangle	.30	.19
Quadrilateral	1.12	1.75
Pentagon	1.19	.94
Acute Angle	1.22	1.50
Right Angle	1.51	1.06
Obtuse Angle	.82	.63

TABLE 5 -- Continued

Statistic	Predicted	Observed
Alternations of success and failure:		
Triangle	3.58	4.00
Quadrilateral	9.37	8.94
Pentagon	9.68	9.06
Acute Angle	10.95	10.25
Right Angle	11.59	10.75
Obtuse Angle	8.16	7.13
Expected pairs of errors on trials n and n + 1:		
Triangle	.47	.25
Quadrilateral	2.75	3.00
Pentagon	3.09	3.43
Acute Angle	2.61	3.00
Right Angle	5.33	5.68
Obtuse Angle	1.53	2.06
Expected pairs of errors on trials n and n + 2:		
Triangle	.43	.56
Quadrilateral	2.62	2.43
Pentagon	2.94	3.18
Acute Angle	2.51	2.50
Right Angle	5.10	5.93
Obtuse Angle	1.46	1.68

TABLE 5 -- Continued

Statistic	Predicted	Observed
Net difference between SE and EE (error following success and error following error):		
Triangle	1.21	1.63
Quadrilateral	1.74	1.38
Pentagon	1.55	.94
Acute Angle	2.69	2.00
Right Angle	.22	.50
Obtuse Angle	2.41	1.38

TABLE 6

PROBABILITY OF A SUCCESS FOLLOWING A SUCCESS AND OF A SUCCESS FOLLOWING AN ERROR FOR RESPONSES PRIOR TO THE LAST ERROR

Geometric Concept	Prob. of SS	Prob. of ES
Quadrilateral	.63	.58
Pentagon	.63	.55
Acute Angle	.70	.61
Right Angle	.55	.46
Obtuse Angle	.77	.60

TABLE 7

STATIONARITY RESULTS, USING THE PROPORTION OF SUCCESSES FOR EACH QUARTILE BEFORE FINAL ERROR

Geometric Concept	Chi-square	df	P >
Quadrilateral	1.75	3	.60
Pentagon	1.33	3	.70
Right Angle	.95	3	.80
Obtuse Angle	12.63	3	.001*
Acute Angle	16.43	3	P < .001*

*Significant

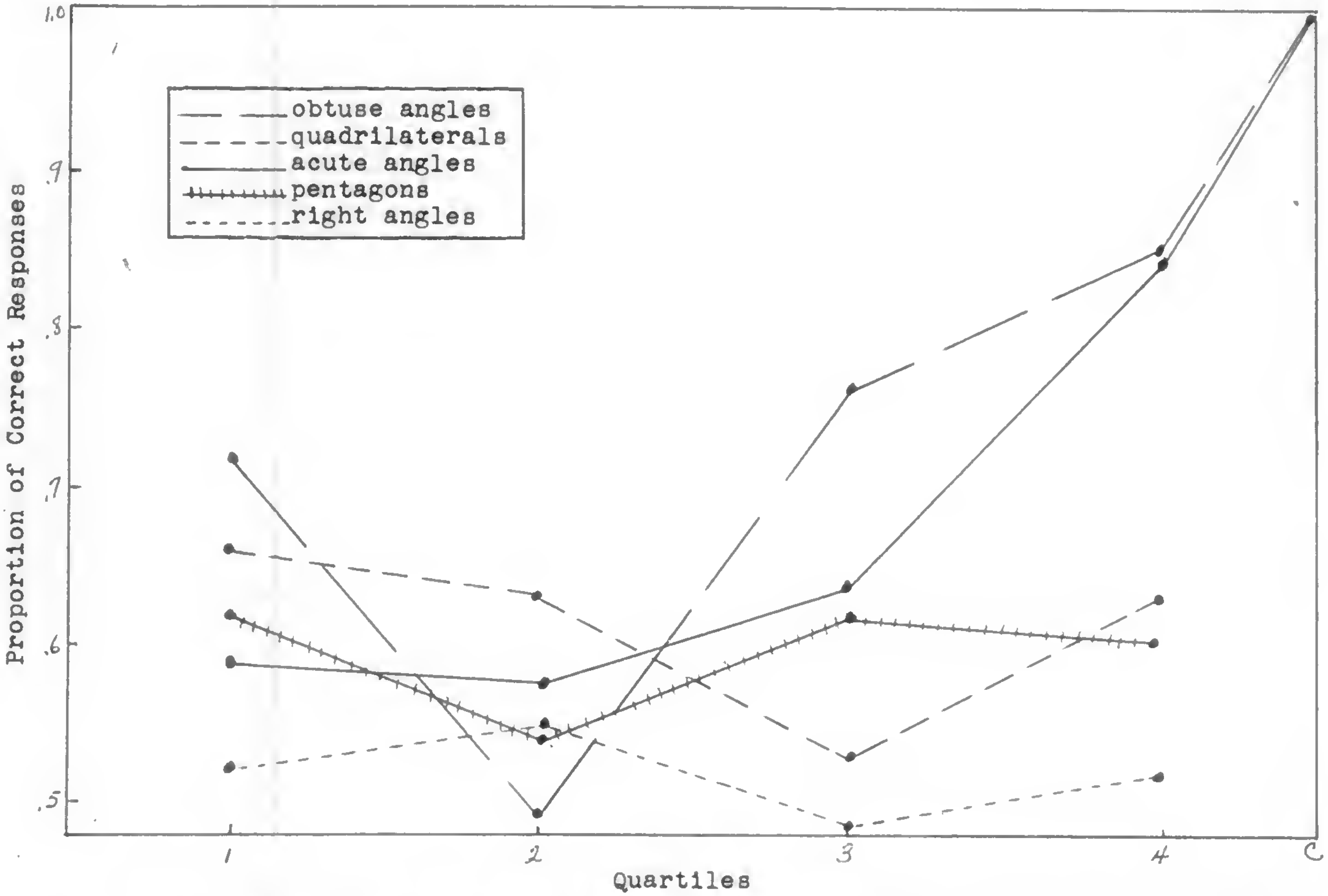

Fig. 22. Vincent learning curves in quartiles for proportion of correct responses prior to the last error for five concepts.

CHAPTER IV

DISCUSSION OF RESULTS

A. The Effect of Models on Learning Rate

The results of the experiment support the hypothesis that the speed with which kindergarten children learn concepts of geometric form and angle size in a situation employing immediate reinforcement and overt correction, but no formal instructions, is related to the number of models present for the subject at the time the stimulus is presented. Tables 1 and 2 summarize the analysis of the data presented graphically in Figures 1 and 2. Even during the first block of nine trials the three-model subjects make more correct responses than the no-model subjects. The reliability of this difference is enhanced over the total 23 blocks; e.g., in the first block for the polygon groups, the two-model and three-model groups are significantly different at $P < .04$; after 23 blocks the two-model and three-model groups are significantly different at $P < .001$. There is no significant difference, however, between the group having no model figure and the group having one model figure. This result obtains considering either the first block of nine trials or all 207 trials of the experiment.

In the polygon groups the probability of correct responding for the one-model group is above that of the no-model group for the last seven blocks. For the angle groups the learning curve for the one-model group is above the no-model group in all blocks except the fourth one. There was one subject in the no-model group for polygons who reached criterion (during the fourteenth block). He was the only subject in both the experimental groups and in the pilot study groups to reach criterion in the no-model conditions. Since there were only eight subjects in each group, this one deviant score raised the mean learning curve for the polygon no-model group considerably.

Considering only the three-model conditions, there appeared to be no significant difference between ease of learning the polygon or the angle problem. But the polygon group reached criterion earlier and was above the angle group in number of correct responses in 13 out of 15 blocks of trials. Possibly the angle problem was more difficult than the polygon problem because the right angle did not vary in size while the acute and obtuse angles did vary. Since angle size was the relevant dimension, this factor of presenting the relevant dimension exactly as the model figure for one concept and not for the others may have increased the difficulty. The data (e.g., Figure 9) indicates that the right angle was the most difficult of the six concepts.

In the three-model conditions the concept learned with the least number of errors was the triangle (Figure 8). It is likely that the subjects may have simplified the polygon problem first on a two-choice basis: either by learning "triangle" and "not triangle" or by learning the general configuration "quadrilateral-pentagon" and "not quadrilateral-pentagon." This type of partial solution might have been less likely if the concepts had been more similar; e.g., if the stimuli had included quadrilateral, pentagon, and hexagon instead of triangle, quadrilateral, and pentagon. Even for the two-model condition (Figure 10) the learning curve for the triangle is more often above the learning curves for the pentagon and quadrilateral than it is below, but there is considerable overlap.

In Figure 11 the curve for the acute angle overlaps, at different points, both of the curves for the other two angle concepts. Except for two points where the curves are tangent, the curve for the obtuse angle is above the curve for the right angle after the first block of trials. The learning curves for the two-model situation (Figures 13 and 14) indicate there were more correct responses for the concept when its model figure was present than when the model for the concept was absent, but the difference was not highly significant. The model-present curve for the angle concepts, however, remains above the no-model-present curve at all points. The curves for the triangle concept (Figure 14) indicates that, under the two-model condition, there is no difference in learning to identify the triangle whether a model of a triangle is present or absent. This finding supports the earlier suggestion that the subjects may have simplified the polygon problem by first making it a two-choice problem.

In the one-model condition (Figures 15 and 16) the learning curves for the model-present concept were not significantly different from the learning curve for the model-absent concepts. However, there was a slight tendency for more correct responses in the polygon group when the model for the concept was present; e.g., there was a slight tendency for those subjects whose model figure was a quadrilateral to make more correct responses in the presence of the quadrilateral stimuli than there was for those subjects whose model figure was a triangle or a pentagon.

B. The Evidence for All-or-None Learning

The results of the experiment support the hypothesis that simple concepts of geometric form and angle size are learned in an all-or-none fashion. Table 3, presenting the results of the goodness-of-fit tests for stationarity, order, and binomial distribution of responses contains only two significant differences: the binomial distribution for the right angles and the order test for the obtuse angles. The stationarity and order tests for the acute angles approach the .05 level of significance, however. The other results support the constant guessing assumption of the all-or-none conditioning model.

Figures 17-21 show the differences between the mean learning curves (1) when all responses are considered and (2) when only the responses prior to the last error are considered. All of the figures, except the one for the acute angle, strongly support the assumption of a constant guessing probability before conditioning.

Although the order tests usually were not significant, a systematic trend is evident in Table 6. It was noted that for all concepts there tended to be a greater probability for success to follow success than for success to follow error. There was considerable variance in the trial number on which subjects made their last error. In view of the tendency for success to follow success, it is possible that there may have been some incremental learning prior to the last error, but it may have been masked in averaging together all subjects without taking into account whether they became conditioned early or late in the series. Those subjects reaching criterion in the early trials may have inflated the guessing probability during the early trials and masked the incremental learning of other subjects, thus providing a false picture of stationarity of responses.

As pointed out by Suppes and Ginsberg (1961), the property of stationarity of response probability prior to the last error is crucial in supporting the assumption that learning occurs in an all-or-none fashion. To take into account possible individual differences, Vincent-type learning curves were constructed showing the proportion of correct responses prior to the last error. These curves were constructed by dividing into quartiles the responses made by each subject with the first data point representing the proportion of correct responses in the first 25 per cent of the responses of all subjects. The data points on the curves are the mean percentiles of the four quartiles: 12.5%, 37.5%, 62.5%, 87.5%. The _C_ which is located one half the distance of a quartile beyond 4 (87.5%) represents criterion or 100% (Suppes and Ginsberg, 1961). Figure 22 presents these data. Table 7 summarizes the results of the chi-square test of the data. Figure 22 indicates the change in form of the curves for the concepts of acute and obtuse angles when drawn using quartiles. The curves for the quadrilateral, pentagon, and right angle remain stationary. But, after the first two quartiles, the curves for the acute and the obtuse angles are concave. This concavity indicates that the simple all-or-none model accounts for about two-thirds to three-fourths of the data for the acute and obtuse angle concepts. Suppes and Ginsberg (1961) suggest that a two-stage model might account for the obtained data where the subject is assumed to show some intermediate level of improvement between his initial guessing level and perfect responding.

It is likely that the subjects first discriminated between the obtuse and acute angles on an all-or-none basis and then gradually acquired the concept of the two angles as different from the right angle. In the polygon problem, the subjects immediately discriminated between the triangle and the other two concepts, simplifying the learning situation so that only two choices had to be considered. The quadrilateral and pentagon concepts were so similar that their attainment was on an all-or-none basis. The one-element model may have accounted for the data here because there were fewer stimuli being considered in the polygon problem than in the angle problem.

Table 5, presenting a comparison of certain predicted and observed data, indicates that in this situation the one-element model does predict a fair approximation to details of the data. There was a large difference, however, between the predicted and observed standard deviation of expected errors per item per S. This difference may have resulted from the curtailment of the distribution (three subjects were replaced because they did not reach criterion).

Despite these differences the overall statistical evidence indicates that the all-or-none conditioning model seems to account for a large portion of the data. The results of tests for the quadrilateral, pentagon, and right angle are especially impressive. However, some other factor seems to be operating in the empirical results for the acute and obtuse angles. Despite the fact that the stationarity tests were not significant, the concave Vincent curves do not support the assumption of constant guessing. It is beyond the scope of this discussion to offer hypotheses concerning the type of model which might be applicable. However, the Suppes and Ginsberg report provides a mathematical basis for further statistical analysis of the data (Suppes and Ginsberg, 1961).

C. Non-systematic Observations

The experimenter noted behavior in the subjects very similar to that described in the learning behavior of primates and animals of lower species. In the no-model and one-model situations where the subject made more errors than correct responses and therefore received little reinforcement, the subjects occasionally resorted to blind guessing, sometimes saying, "Oh, I'll just choose any one." Fatigue was more evident under these conditions, also. In a few cases, when it became evident to a subject that he was continuing to make errors, even after he had examined the stimuli carefully, he expressed a color or position preference and used that particular button for his response key during the remainder of the day's session regardless of the stimulus presented. During the early blocks of trials the subjects exhibited behavior similar to that which Tolman described as "vicarious-trial-and-error" (Hilgard, 1956). This vacillation before responding was particularly evident in the two-model and three-model groups.

In the groups having one-model and two-model figures, some subjects tended to avoid using the model altogether; others tended to avoid classification in the no-model category.

During the early trials many of the subjects attempted to classify the stimuli by the direction in which the most acute angle pointed; e.g., one classification would include all figures with a sharply acute angle pointing to the right. Of those who reached criterion and verbalized about their basis for classification, not one subject mentioned the number of sides. Some talked of shapes such as: "a kind of house top" (triangle), "a crooked door" (quadrilateral), "a funny birdhouse" (pentagon). Others talked in terms of the number of "points" (vertices). In classifying angles, those subjects who reached criterion referred to them in such terms as "skinny" (acute), "fat" (obtuse), and "in-between" (right). During the early trials such comments as: "an _A_," "an upside-down _A_," or a "house-top" were made frequently.

It was never clear, however, whether a subject formed his concept basically from noting a configuration or by noting specific characteristics of the stimuli. It was not possible to conclude whether the subject had used the experimenter-defined dimension, particularly for the polygon.

D. Summary

This experiment tested the hypotheses that (1) the rate at which kindergarten children learn concepts of geometric form and angle size is related to the number of representative models present at the time the stimulus is presented and (2) simple concepts of geometric form and angle size are learned in an all-or-none fashion. Eighty-eight kindergarten pupils were exposed to a series of 153 plane outline figures representing three concepts: form concepts (triangle, quadrilateral, and pentagon) and angle concepts (acute angle, right angle, and obtuse angle). The subjects were divided into eight groups (four for each class of concepts -- form or angle) which operated in four conditions: no-model, one-model, two-model, and three-model representations. There were 16, 16, 24, and 32 children in each condition (half exposed to the form problem and half exposed to the angle problem). The results of the experiment supported the hypothesis that the rate at which these concepts are learned by kindergarten children is related to the number of representative models present at the time the stimulus is presented.

To test the hypothesis that simple concepts of geometric form and angle size are learned in an all-or-none fashion, the data for the three-model group were analyzed, using the responses prior to the last error. The following assumptions were tested: (1) no learning takes place before the last error, (2) responses are statistically independent from trial to trial, (3) a subject guesses at random before the last error. The results of these tests supported the hypothesis with only two exceptions: the binomial distribution for the right angle and the independence of response test for the obtuse angle. Since the data indicated that there was a tendency for success to follow success more frequently than for error to follow success, it was felt that individual differences in incremental learning might be masked by averaging over the responses of all subjects without taking into account the percentile in which the subject's responses occurred. Vincent-type curves were constructed by dividing each subject's responses into quartiles averaging over subjects within the quartiles. The Vincent curves were stationary for the quadrilateral, pentagon, and right angle concepts but showed increasing guessing probability for the obtuse and acute angle concepts. These results indicate that data for three of the concepts can be accounted for adequately by the one-element model but some sort of incremental model is necessary for the other two concepts. Suppes and Ginsberg (1961) have proposed a two-stage model which may account for these differences.

BIBLIOGRAPHY

Bower, G. H. "Application of a Model to Paired Associate Learning," Psychometrika, vol. 26, pp. 255-80 (1961).

Hilgard, E. R. Theories of Learning. New York: Appleton-Century Crofts, Inc., 1956.

McNemar, Quinn. Psychological Statistics. New York: John Wiley and Sons, Inc., 1955.

Suppes, P. and Ginsberg, R. Application of a Stimulus Sampling Model to Children's Concept Formation of Binary Numbers, with and without an Overt Correction Response. Technical Report #35, Psychology Series, Institute for Mathematical Studies in the Social Sciences, Applied Mathematics and Statistics Laboratories; Stanford University, 1960. (in press, Journal of Experimental Psychology).

Suppes, P. and Ginsberg, R. A Fundamental Property of All-or-none Models, Binomial Distribution of Responses Prior to Conditioning, with Application to Concept Formation in Children. Technical Report #39, Psychology Series, Institute for Mathematical Studies in the Social Sciences, Applied Mathematics and Statistics Laboratories; Stanford University, 1961.

Vinacke, W. E. The Psychology of Thinking. New York: McGraw-Hill Book Company, Inc., 1952.

APPENDIX

CONSTRUCTION OF STIMULI

The stimulus figures were black outlines 1/8" wide, drawn on white cards 8½" x 11". All figures, both polygons and angles, were constructed using three squares (3", 4", and 5") and two rectangles (3½" x 4½", 3" x 4") as basic guides for the stimulus area; i.e., the polygons and the angles were inscribed within the basic figures.

In the polygon group there were fifty-one each of the triangles, quadrilaterals, and pentagons. In the angle group there were fifty-one each of the acute angles, right angles, and obtuse angles. The triangles were divided into five groups: right, isosceles, equilateral, scalene acute, scalene obtuse. In each of these groups there were ten variations, except in the scalene acute, where there were eleven in order to take care of the fifty-first one. From a random number table, each group of triangles was randomized for size with two triangles of each group being cut from each of the basic figures, one figure being used three times for the scalene acute. The triangles were rotated so that no two figures had the same area, rotation, and specific shape.

The quadrilaterals were divided into six groups: five squares, five rectangles, five rhombuses, five parallelograms, ten trapezoids, twenty-one trapeziums. The same basic figures were used to determine the area. From each size were cut one square, one rectangle, one rhombus, one parallelogram, two trapezoids, four trapeziums, with one size yielding five trapeziums. The pentagons were divided into two groups: five regular pentagons and forty-six irregular pentagons. One regular pentagon and nine irregular pentagons were cut from each basic figure, one size being used for ten irregular pentagons. Both quadrilaterals and pentagons were randomly rotated so that rotation could not be used as a relevant cue for correct classification.

The acute and obtuse angles were randomized in a manner similar to the triangle randomization. There were five groups of acute angles: 15°, 30°, 45°, 60°, 70°, and five groups of obtuse angles: 110°, 120°, 135°, 150°, 165°. Each of the groups was rotated through ten different positions: 0°, 45°, 60°, 90°, 135°, 180°, 225°, 240°, 270°, 315°, the fifty-first angle being rotated through 300°. The length of the sides were determined from the five basic figures to which they were randomly assigned. The right angles were rotated through fifty-one different positions (all positions ending in 0° -- from 0° through 350° -- 15°, 45°, 75°, 85°, 115°, 135°, 165°, 185°, 215°, 225°, 265°, 275°, 305°, 315°, 345°). The length of the sides were again determined by random assignment to one of the basic figures.

Order of presentation of the stimuli: The stimulus cards were numbered as follows: (1) polygons 1-153 (triangles 1-51, quadrilaterals 52-102, pentagons 103-153), (2) angles 154-306 (acute angles 154-204, right angles 205-255, obtuse angles 256-306). From a table of random numbers the cards were randomized in blocks of nine (three of each concept in each set being assigned to a block). There were sixteen randomizations for the polygon set and sixteen for the angle set. Each of the first eight randomizations was presented to four subjects: one from each of the conditions of model representation. Each of the next four randomizations was presented to two subjects: one from the two-model group and one from the three-model group. Each of the other four randomizations was presented to only one subject in the three-model group.

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